

PHYSICAL AND ELECTRICAL ENGINEERING
LABORATORY MANUALS

EXAMPLES IN
Electrical Engineering

UC-NRLF



B 3 141 173

Mech. Dept.

LIBRARY

OF THE

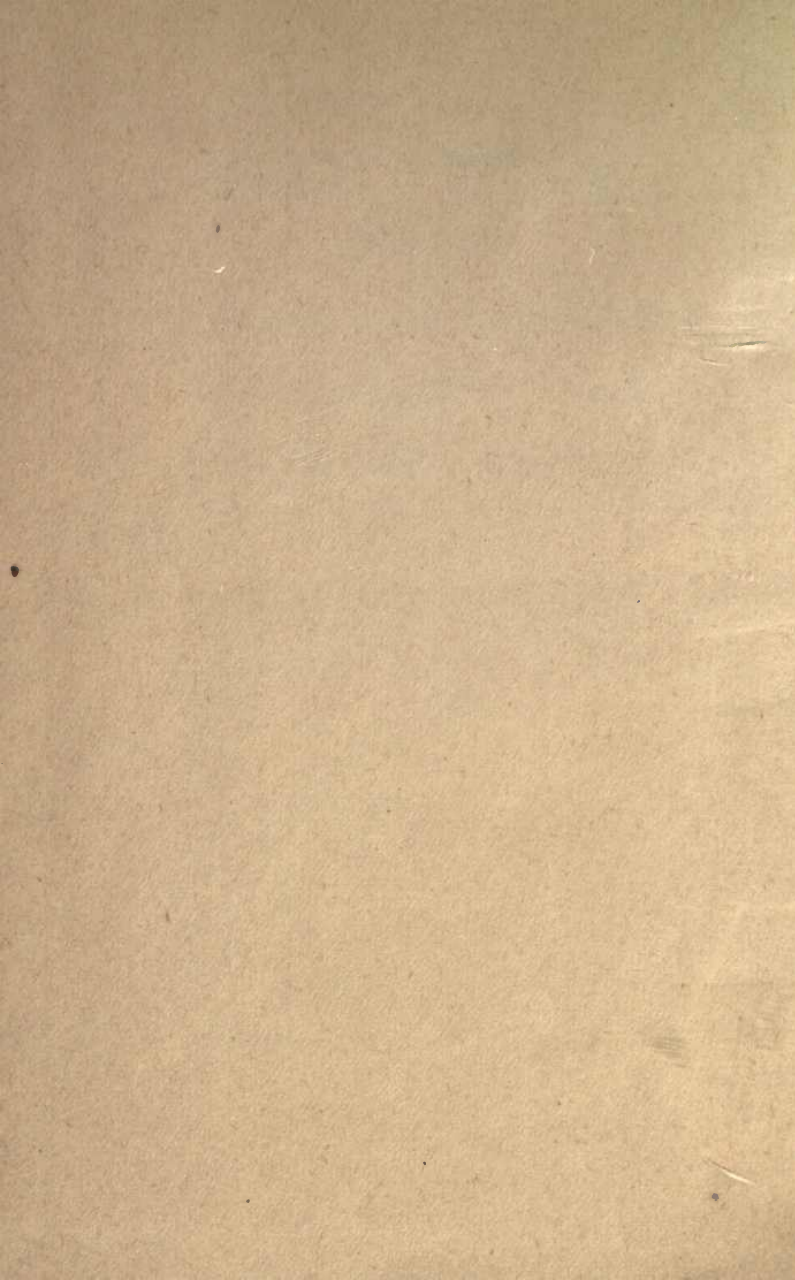
UNIVERSITY OF CALIFORNIA.

Received *Sept*, 189*8*

Accession No. *74023* . Class No.

Engineering
Library

Library of the Department
of
Mechanical and Electrical Engineering



EXAMPLES IN
ELECTRICAL ENGINEERING

*PHYSICAL AND ELECTRICAL ENGINEERING
LABORATORY MANUALS.—Vol. 1.*

ELEMENTARY PHYSICS.

By JOHN HENDERSON, B.Sc. (Edin.), A.I.E.E., Lecturer
in Physics, Manchester Municipal Technical School.

Crown 8vo, 2s. 6d.

LONGMANS, GREEN, & CO.
LONDON, NEW YORK, AND BOMBAY.

EXAMPLES IN ELECTRICAL ENGINEERING



BY

SAMUEL JOYCE, A.I.E.E.

LECTURER TO THE SENIOR CLASSES IN ELECTRICAL ENGINEERING
MUNICIPAL TECHNICAL SCHOOL, MANCHESTER



LONGMANS, GREEN, & CO.

LONDON, NEW YORK, AND BOMBAY

1896

All rights reserved

TK168

J6

Engineering
Library

74023

P R E F A C E

MANY of the examples in the following pages have been collected during the past few years to illustrate the author's lectures to Advanced and Honours Students in Electrical Engineering, though the majority are here published for the first time.

Originally intended as a collection of exercises, the explanatory matter forming the bulk of the text was, however, found necessary to make the book more complete in itself, though it is not intended to act as a full treatise on the subject. These explanations, together with the tables at the end of the book, will, it is hoped, be found very useful by draughtsmen and others engaged in electrical machine design.

The author's best thanks are due to such writers as have been made use of, too numerous to mention by name ; and also to two of his third-year students, Messrs. A. B. Mallinson and W. K. Meldrum, for many carefully executed diagrams.

Lastly, and not the least, the author's thanks are due to his friend Mr. E. S. Shoults, for considerable assistance in checking examples.

S. JOYCE.

LATCHFORD HOUSE, GREENHEYS,
MANCHESTER,
July, 1896.

NOTE.—The author will be much obliged if readers will kindly notify any errors that may have escaped observation.

CONTENTS

| CHAPTER | PAGE |
|---|------|
| I. SIMPLE CIRCUITS | 1 |
| II. DISTRIBUTION. MULTIPLE WIRE CIRCUITS | 15 |
| III. CIRCUITS CONTAINING MORE THAN ONE E.M.F. | 27 |
| IV. DYNAMOS AND MOTORS | 35 |
| V. ARMATURE WINDING. | 55 |
| VI. THE MAGNETIC CIRCUIT | 65 |
| VII. MAGNET COIL WINDINGS | 82 |
| VIII. ARMATURE REACTIONS | 86 |
| IX. EFFICIENCY | 94 |
| X. VOLTMETERS AND THEIR RESISTANCE COILS. | 102 |
| XI. ELECTRIC TRACTION, RAILWAYS, ETC. | 106 |
| XII. ALTERNATING CURRENT CIRCUIT | 113 |
| XIII. IMPEDANCE COILS AND TRANSFORMERS | 145 |
| XIV. EFFECTS OF CAPACITY | 160 |
| XV. REGULATING RESISTANCES | 174 |
| XVI. PRIME MOVER COSTS | 181 |
| EXAMPLES | 186 |
| ANSWERS | 205 |
| TABLES | 209 |
| INDEX. | 236 |





EXAMPLES IN ELECTRICAL ENGINEERING.

CHAPTER I.

SIMPLE CIRCUITS.

The Simple Electric Circuit consists of a conducting path, insulated so as to confine the electric flux as much as possible to that path. In it the relationship known as Ohm's Law holds good, viz.—

$$C = \frac{e}{R}$$

If e be stated in volts, R in ohms, then C will be in ampères. The value of e in the above expression must be considered as being the algebraic sum of all the electro-motive forces (E.M.F.) acting in the circuit, and will be called the effective or active E.M.F.

The resistance of a simple circuit may be composed of a number of separate resistances in series; in which case R will be the sum of all these other resistances, thus—

$$R = r_1 + r_2 + r_3 + r_4$$

$$\text{and therefore } C = \frac{e}{r_1 + r_2 + r_3 + r_4}$$

$$\text{or } e = Cr_1 + Cr_2 + Cr_3 + Cr_4$$

and each of these quantities may be called a potential difference, or P.D., for short. The E.M.F. e is thus the sum of all the P.D.'s, or is equal to—

$$\Sigma Cr$$

writing the P.D. $Cr_1 = v_1$, we have that—

$$e = v_1 + v_2 + v_3 + v_4$$

The Rate of doing Work.—The amount of work done

in a circuit is equal to the quantity, Q , of electricity passed multiplied by the E.M.F. And since Q is the product of the current and the time, then the work done, W , in the time t will be—

$$W = eCt$$

from which the rate of doing work, or power, or activity, is—

$$P = \frac{W}{t} = \frac{eCt}{t} = eC$$

and as $e = Cr_1 + Cr_2 + Cr_3 + Cr_4$, etc., we have—

$$P = eC = C^2r_1 + C^2r_2 + C^2r_3 + C^2r_4, \text{ etc.}$$

which gives the distribution of energy expenditure round the circuit, and states that the whole rate of doing work in a circuit is equal to the sum of all the activities in the separate parts of the circuit.

The activity or power P is measured in watts, one watt being the activity when the product eC is unity. Owing to the values chosen for the volt and the ampère, the watt is equivalent to the $\frac{1}{746}$ part of the mechanical horse-power, or—

$$\text{H.P.} = \frac{\text{watts}}{746}$$

Calculation of Resistance.—The value of the resistance of a circuit is determined by the nature of the material of which that circuit is composed, by its temperature, and by the dimensions of the circuit; the relationship between these quantities being that—

$$R = \frac{L\rho}{a}$$

where L and a are respectively the length and sectional area (average if not uniform) measured in terms of the same unit, and ρ represents the resistivity at a certain temperature of the material; that is to say, ρ is the actual resistance in ohms of the unit cube of the material.

The term resistance has come more into general use than conductance, and represents the opposite idea, the relationship between them being—

$$\rho = \frac{1}{m}$$

where m stands for the conductivity, that is the actual conductance, in mhos of the unit cube.

Thus, the value of the resistance of a circuit can be expressed in two ways, viz.—

$$R = \frac{L\rho}{a} = \frac{L}{am} = \text{ohms}$$

and similarly the conductance may also be written—

$$K = \frac{am}{L} = \frac{a}{L\rho} = \text{mhos}$$

Thus, if the dimensions of a circuit, both electrical and mechanical, be known, its resistance or conductance can at once be found. And in cases where the whole resistance is made up of a number of items, we can find the value of each item r by inserting in the equation—

$$r = \frac{l\rho}{a}$$

the proper values for l , a , and ρ , the latter being possibly different for each item of the circuit, either on account of temperature or on account of difference of material.

The tables on page 209 will give all necessary data for these values.

Different Kinds of Circuits.—There are two main ways in which lamps are connected to form a circuit: the series and the parallel.

The *series* circuit is arranged thus—

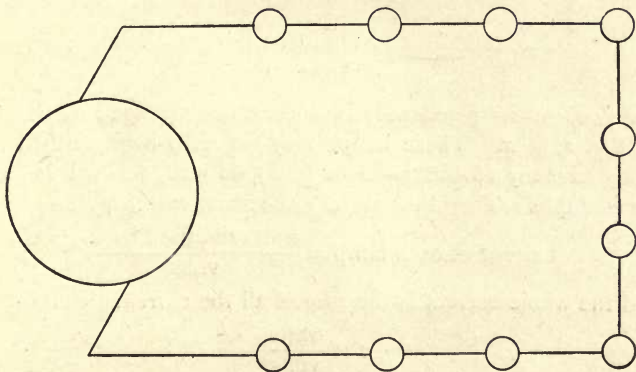


FIG. 1.

and the lamps are generally arcs, each taking such and such a current at a P.D., usually from 45 to 50 volts in the case of continuous currents.

The current in the circuit is that taken by any lamp, and is constant; that is, fixed in value for that particular circuit. The E.M.F. is that required by each lamp multiplied by the number of lamps and added to the P.D. required for the dynamo and leads.

The *parallel* circuit is arranged thus—

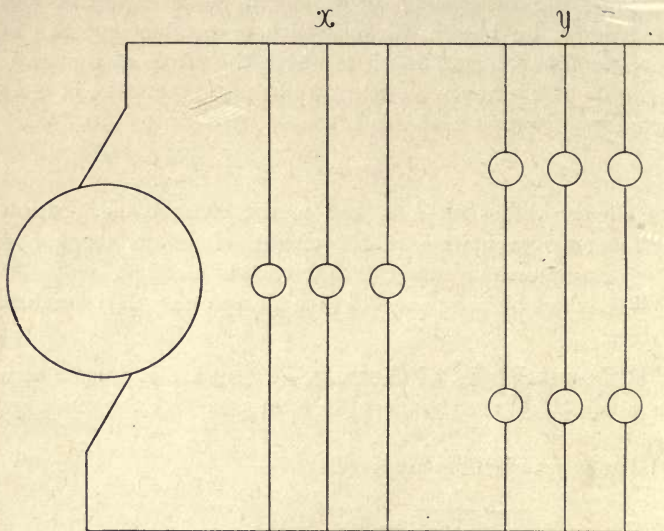


FIG. 2.

and the lamps are generally incandescents, arranged as shown in Fig. 2, at *x*. These lamps may generally be described as being so many 16 candle-power (c.p.), 60 watt, 100 volt lamps, for example. They thus all take the same current, viz.—

$$\text{current in each lamp} = \frac{\text{watts required by it}}{\text{volts}}$$

and the whole current is the sum of all the currents, or is—

$$n \times \frac{\text{watts}}{\text{volts}}$$

In some cases there are arc lamps arranged two in series as shown at *y*, Fig. 2. These lamps will be 10 ampère lamps, for example, and the current in that case will be $10 \times$ half the number of lamps.

Calculation of Resistance of Parallel Circuit.—In considering the value as to resistance or conductance of a number of items in parallel, it is generally simplest to take the sum of all the values of current in ampères through each item of the parallel circuit due to a common P.D. For example, What is the resistance of four lamps in parallel having resistances respectively 100 ohms, 50 ohms, 25 ohms, and 10 ohms? Consider the common P.D. to be, say, 100 volts, whence the currents respectively will be 1 ampère, 2 ampères, 4 ampères, and 10 ampères, making a total of 17 ampères, which current is due to a P.D. of 100 volts acting upon the united resistances in parallel, which must consequently have the value $R = \frac{E}{C} = \frac{100}{17} = 5.88$ ohms.

The direct calculation of conductance is simpler, since the total conductance is obviously the sum of all the conductances. Thus, the respective conductances are 0.01, 0.02, 0.04, and 0.1 mho, or the total conductance is 0.17 mho. Now, conductance is the reciprocal of resistance, and resistance is the reciprocal of conductance, or—

$$K = \frac{1}{R}. \quad \text{And } R = \frac{1}{K} = \frac{1}{\frac{1}{R}}$$

But the total conductance is the sum of all the separate conductances, or is $K = k_1 + k_2 + k_3 + k_4$, etc., which can be replaced by $K = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4}$, etc. And as the resistance is the reciprocal of conductance, we have—

$$R = \frac{1}{K} = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4}}$$

which may be expressed in words: "The resistance of a number of items in parallel is equal to the reciprocal of the sum of the reciprocals of their separate resistances."

In the parallel or multiple-parallel system of distribution the circuit is always composed of a number of items in series, such as the resistances respectively of the dynamo, the conducting wires, and the lamps; and as these latter generally have to work at a fairly constant P.D., notwithstanding variations in the current, it becomes a question of great importance to know what size the leads must be to prevent more than a certain small loss of pressure in them. As a good compound dynamo gives a fairly constant P.D. at its terminals for all values of the current, its own resistance may be disregarded, and the terminal P.D. taken as the whole E.M.F. of the outside circuit. A few typical cases of this kind will render this clear.

✓ **Example I.**—What must be the sectional area in square inches of the copper leads to convey the current required by 200 lamps in parallel, all situated at the far end of the leads, each lamp taking 60 watts at 100 volts? The distance from the dynamo to the lamps is 100 yards, and the loss along the leads is not to exceed 2 volts.

Solution.—Since the loss in the leads may be 2 volts, and is equal to the current multiplied by the resistance of the leads, *i.e.* the P.D. required to send the current through the leads, or Cr_l , it is necessary first to find the total current. Each lamp is a 60 watt, 100 volt, and therefore takes $\frac{60}{100} = 0.6$ ampère; and the whole current is consequently 0.6×200 , or = 120 ampères. Therefore the P.D. required by the leads is 120*r_l*, and must not exceed 2 volts; or—

$$r_l = \frac{2}{120} = 0.016\bar{6} \text{ ohm}$$

which is the resistance of a certain length of wire, *viz.* 2×100 yards. We can write this resistance also in terms of its length, sectional area, and specific resistance, or—

$$r_l = \frac{2 \times 100 \times 36 \times 0.00000067}{a}$$

where a is the sectional area required in square inches, and—

$$a = \frac{2 \times 100 \times 36 \times 0.00000067}{0.0166} = 0.2895 \text{ sq. inch}$$

which would be slightly larger than a $\frac{19}{10}$ stranded cable.

Example II.—If in the above question the resistance of the dynamo be 0.013 ohm, what will be the total E.M.F. in the whole circuit, the P.D. at the dynamo terminals, and the P.D. at the lamps at full load and at half load, when the lamps are supposed to vary one volt up or down, from 100 volts as an average?

Solution.—Neglecting the slight difference made in the total current when the P.D. at the lamp terminals varies slightly, and the shunt current of the dynamo, we have that at full load $C = 120a$, loss in leads = 2 volts, P.D. at lamps is 99 volts, and E.M.F. total must be equal to the sum of these plus the P.D. required to send the current through the dynamo. And this last is 120×0.013 , or 1.56 volt. Whence $E = 99 + 2 + 1.56 = 102.56$ volts.

Again, at half load only 100 lamps are in circuit, and $C = 60$ ampères; whence the loss in leads is only $60 \times 0.01666 = 1$ volt, and the P.D. required for the dynamo resistance is $60 \times 0.013 = 0.78$ volt. The whole E.M.F. is then the sum of these, viz. 1.78, plus the P.D. at the lamp terminals, which is now 100 volts, and total E.M.F. = 101.78 volts. Also the P.D. at the dynamo terminals is in each case the sum of the P.D.'s required by the leads and the lamps. At full load this is $2 + 99 = 101$ volts, and at half load $1 + 100$, or, again, 101 volts. Thus the P.D. at the dynamo terminals is to be kept constant, at 101 volts, for all loads.

Example III.—What will be the diameter of the copper rods to convey 5000 ampères to an aluminium furnace at a distance of 7 yards from the dynamo terminals, if the loss along the leads is to be only $\frac{1}{8}$ volt?

Solution.—The P.D. required by the leads is 0.125 volt, and is equal to $5000r$, whence—

$$r_1 = \frac{0.125}{5000} = 0.000025 \text{ ohm}$$

and the leads are $2 \times 7 \times 36 = 504$ inches long, or have a resistance of—

$$\frac{0.000025}{504} = 0.000000496 \text{ ohm}$$

per one inch. Therefore their sectional area will be—

$$\frac{0.00000067}{0.000000496} = 13.5 \text{ sq. inches}$$

and if circular bars, their diameter will be—

$$\begin{aligned} d &= 2 \times \sqrt{\frac{a}{\pi}} = 2 \times \sqrt{4.3} \\ &= 2 \times 2.073 = 4.146 \text{ inches} \end{aligned}$$

Example IV.—What will be the loss of pressure at half load and full load in the leads in a parallel circuit running 40 arc lamps arranged two in series, and taking 12 ampères each, when the dynamo is kept at a terminal pressure of 102 volts, and is 50 yards from the lamps, the sectional area of the leads being 0.3 sq. inch?

Solution.—At full load the current will be—

$$C = 12 \times \frac{40}{2} = 240 \text{ ampères}$$

and at half load $C = 120$ ampères. The leads are $50 \times 2 \times 36 = 3600$ inches long, and therefore have a resistance—

$$\begin{aligned} r_1 &= \frac{L\rho}{a} = \frac{3600 \times 0.00000067}{0.3} \\ &= 0.00804 \text{ ohm} \end{aligned}$$

Therefore the loss of pressure at full load will be $0.008 \times 240 = 1.92$ volt, and at half load $0.008 \times 120 = 0.96$ volt nearly.

In simple series circuits there is no variation in the loss of pressure in the leads, etc., with variation of load since the current is constant. It is, then, only necessary to keep the loss of energy in the leads within a reasonable percentage of the whole activity. Thus in the following examples:—

Example V.—A series circuit consists of 60 ten-ampère arc lamps, spaced 100 yards apart, and taking an average of 49 volts each. What will be the diameter of the single copper leads required in order that the energy waste in them will be within $2\frac{1}{2}$ per cent. of the energy required by the lamps?

Solution.—The expenditure of energy in the lamps is at the rate of $60 \times 49 \times 10 = 29,400$ watts, and $2\frac{1}{2}$ per cent. of this is $\frac{2.5}{100} \times 29,400 = 735$ watts, which is a loss of 73.5 volts, as the current is 10 ampères, and consequently their resistance is 7.35 ohms. The length of the leads is $60 \times 100 \times 36 = 216,000$ inches long, and thus the resistance per inch is—

$$\frac{7.35}{216,000} = 0.000034 \text{ ohm}$$

Thus the sectional area is $\frac{0.00000067}{0.000034} = 0.0197$ sq. inch, whence the diameter will be—

$$\begin{aligned} \text{diameter} &= 2 \times \sqrt{\frac{a}{\pi}} = 2 \times \sqrt{0.00626} \\ &= 0.158 \text{ inch} \end{aligned}$$

or about No. 8 B.W.G.

Example VI.—What will be the brake H.P. of engine required to run the above lamps if the loss in the dynamo itself is 150 volts, and only $\frac{9}{10}$ of the power received is converted into electricity?

Solution.—The activity in the lamps is $60 \times 49 \times 10 = 29,400$ watts, and the loss in the leads is 735 watts. The loss in the dynamo is $10 \times 150 = 1,500$ watts; whence the whole activity is 31,635, or the electrical H.P. is—

$$= \frac{31635}{746} = 42.41$$

But this is only $\frac{9}{10}$ of the H.P. at the engine pulley, whence the brake H.P. of engine must be—

$$42.41 \times \frac{10}{9} = 47.13 \text{ H.P.}$$

Example VII.—What will be the E.M.F. required to send a current of 10 ampères through a circuit consisting of a dynamo whose resistance is 7 ohms, of $1\frac{1}{2}$ mile of leads of No. 6 B.W.G. = 0.203 inch diameter, and 80 lamps each of 2.5 ohms resistance? What will be the H.P. of engine required if the dynamo turns $\frac{9}{10}$ of the power received into electricity? And what will be the cost of 1 Board of Trade unit in the lamps if 1 mechanical H.P. cost twopence per hour?

Solution.—Total resistance of circuit is—

$$7 + 80 \times 2.5 + \text{leads}$$

and the resistance of the leads is—

$$r_l = \frac{1.5 \times 1760 \times 36 \times 0.00000067}{0.203 \times 0.203 \times 22} = 4 \times 7$$

$$= 1.96 \text{ ohm}$$

or whole resistance is 208.96 ohms ;

whence the E.M.F. is $10 \times 208.96 = 2089.6$ volts

and the total activity is $2089.6 \times 10 = 20,896$ watts

and therefore the engine power must be—

$$20,896 \times \frac{10}{9} \times \frac{1}{746} = 31.12 \text{ H.P.}$$

Thus the cost per hour's run will be 31.12 H.P. at 2d., or 62.24d., which has to be paid for by the charge for the lamps, or is the cost of $200 \times 10 \times 10$ watt-hours, or of 20 B.T.U.; whence the cost—

$$= \frac{62.24}{20} = 3.112d. \text{ per B.T.U.}$$

Insulation Resistance.—The insulation resistance of a circuit, *i.e.* the resistance between any conductor and earth, is measured for convenience in megohms, one megohm being 1,000,000 ohms. As far as insulation goes, all parts of a circuit are in parallel, and if R be the resistance of any one part to earth, then the resistance of a circuit consisting of n such parts, whether joined electrically in series or parallel, will only be $\frac{1}{n}$ of R. Fuse blocks, joints, fittings, switches, and

connections generally are all sources of leakage, and usually bring down the insulation resistance very considerably, in spite of the fact that the resistance to earth of the cable employed may be many thousands of megohms.

Example VIII.—A circuit 500 yards long is made of cable having an insulation resistance of 3000 megohms per mile, and has 20 joints in it, each of 10,000 megohms resistance, and 250 fittings of 800 megohms each. What is the total insulation resistance?

Solution.—All the above items are in parallel, but as the cable is only 500 yards long at 3000 megohms per mile, the resistance of this item will be—

$$3000 \times \frac{1760}{500} = 10560 \text{ megohms}$$

The 20 joints in parallel will be $\frac{1}{20} \times 10000 = 500$ megohms collectively, and the 250 fittings will together be—

$$\frac{1}{250} \times 800 = 3.2 \text{ megohms}$$

The whole insulation resistance is consequently all these in parallel, or will be—

$$\frac{1}{\frac{1}{10560} + \frac{1}{500} + \frac{1}{3.2}} = 3.1 \text{ megohms}$$

being practically that due to the fittings alone, and quite independent of the quality of cable, the high insulation of which must, however, be regarded as an indication of its durability.

Example IX.—A central station makes a rule that the insulation resistance of any consumer's installation must not be less than 0.2 megohm. If there are 600 consumers, and the P.D. between the mains and earth is 100 volts, what will be the total leakage current?

Solution.—The total insulation will be $\frac{1}{600} \times 0.2 = 333$ ohms, whence the leakage current is $\frac{100}{333} = 0.3$ ampère.

In the measurement of activity by means of an ammeter

and a voltmeter, connections may be made in two ways—

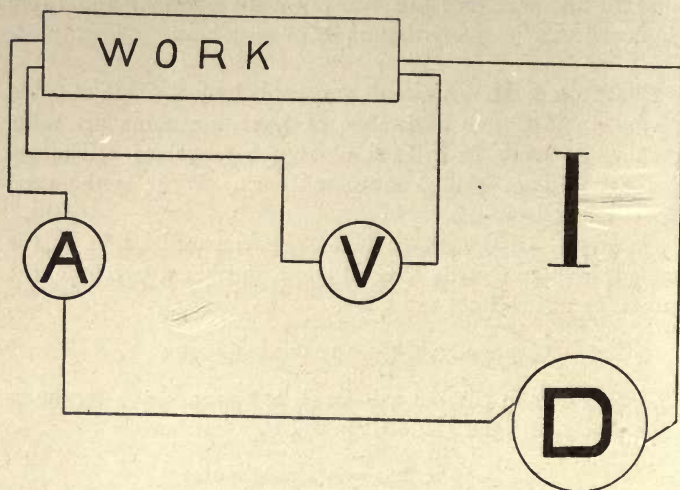


FIG. 3.

or—

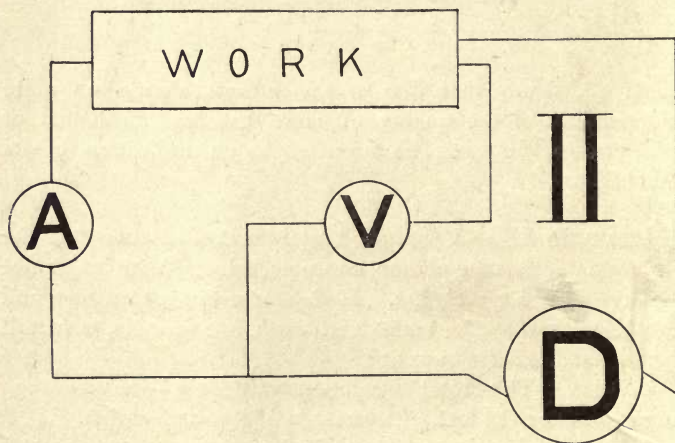


FIG. 3a.

In Case I., if the voltmeter be not of the electro-static type, the ammeter reads the current in the work added to the current in the voltmeter when simultaneous readings are taken; whilst the voltmeter reads the true P.D. at the terminals of the work.

Let R = resistance of voltmeter.

r = „ ammeter.

e = true P.D. at terminals of work.

C = „ current in work.

Then the current in the voltmeter will be $\frac{e}{R}$, and the ammeter reads $C + \frac{e}{R}$, whilst the voltmeter reads e .

The product of these two is $e \left(C + \frac{e}{R} \right) = ec + \frac{e^2}{R}$, and is the apparent activity in the work. But the true activity is obviously ec , and the difference is easily recognized as the power lost in the voltmeter. Thus the measured power is the true power plus the power required to actuate the voltmeter.

In Case II. similarly the measured power is the sum of the true power plus the power required to work the ammeter. It is thus necessary, in readings of this sort, to make corrections, especially when the activity to be measured is comparable in magnitude to the power required to work the instruments used. For example—

Example X.—It is required to know how many watts a certain incandescent lamp requires, connections being made as in Case I. (Fig. 3). The ammeter indicates a current of 0.66 ampère, whilst the voltmeter reads 101 volts, and has a resistance of 2000 ohms.

Solution.—Since 101 volts is the P.D. at the terminals, the current in the voltmeter is—

$$\text{Current in voltmeter} = \frac{101}{2000} = 0.0505 \text{ ampère}$$

and the reading of the ammeter is 0.66 ampère; therefore the current in the lamp is $0.66 - 0.0505 = 0.6095$ ampère. Thus the total activity is—

$$\text{Activity} = 0.6095 \times 101 = 61.5595 \text{ watts}$$

Connections as in Case II. (Fig. 3) do not give such reliable information as those in Case I., for the reason that in ascertaining the loss of volts in the ammeter its resistance must be accurately known; and as this is low and partly made up of variable contact resistances at the terminals, there is generally considerable uncertainty as to its exact value. An example is, however, given.

Example XI.—What is the resistance of a lamp when the ammeter reads 0·67 ampère and the voltmeter indicates 101 volts? Connections are made as in Case II. (Fig. 3), and the resistance of the ammeter is 0·75 ohm.

Solution.—The actual current in the lamp is given by the ammeter as 0·67 ampère, and consequently the loss of volts in the ammeter is $0·67 \times 0·75 = 0·5025$ ampère, and thus the P.D. at the terminals of the lamp will be—

$$101 - 0·5025 = 100·4975 \text{ volts}$$

$$\text{from which its resistance is } \frac{100·4975}{0·67} = 150 \text{ ohms}$$

Note.—Power-meters and ohm-meters, which are virtually combinations of an ammeter with a voltmeter arranged so as to give, in the one case, the product of volts and ampères, and in the other case, the ratio of volts to ampères, can, in calibration, be corrected for errors due to the current in the voltmeter part, or P.D., required by the ammeter coil, but must be then connected up in such a way as to agree with such correction.

CHAPTER II.

DISTRIBUTION.

MULTIPLE WIRE CIRCUITS.

So far the examples given have only dealt with cases where the lamps are all connected in a bank at the end of the leads, remote from the generator. In a great many cases, however, the load is more or less uniformly distributed along the length of the circuit. If the arrangement be quite uniform, the case is simple. Consider two parallel mains lead and return, having lamps uniformly spaced between them, as in Fig. 4.

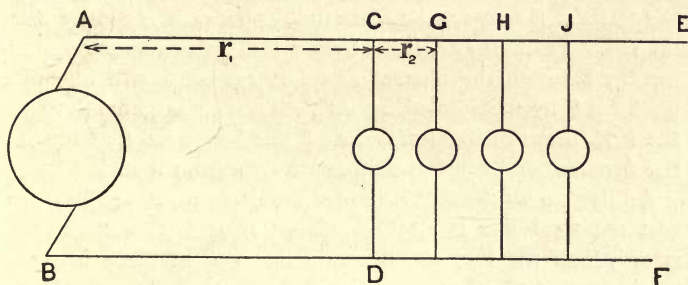


FIG. 4.

Let AB represent the terminals of a dynamo supplying current to a number of lamps, the nearest being situated at a distance AC from the dynamo, and any lamp being at a distance CG from its neighbour. Let the resistance of the leads from A to the first lamp be r_1 ohms lead and return, and the resistance of the lead and return between any two lamps be r_2 ohms. Now, all the lamps obviously cannot be at the same P.D.,

since there will be a loss of pressure in between each lamp and the next nearer the dynamo, of an amount equal to r_2 times the current flowing to all the lamps beyond the point under consideration. Thus, if the current taken by each lamp be C ampères, neglecting the slight variation of current due to the fact that all lamps are not at the same P.D., the loss of volts between $CG = r_2 \times 3C$, and similarly the loss between GH and HJ , will be $r_2 \times 2C$, and $r_2 \times C$. There will also be a loss of $r_1 \times 4C$ volts between the terminals of the dynamo and the nearest lamp. If e represents the P.D. at the dynamo terminals, the P.D. at the terminals of the four lamps, C , G , H , and J , will be respectively—

$$e - r_1 \times 4C$$

$$e - r_1 \times 4C - r_2 \times 3C$$

$$e - r_1 \times 4C - r_2 \times 3C - r_2 \times 2C, \text{ and}$$

$$e - r_1 \times 4C - r_2 \times 3C - r_2 \times 2C - r_2 \times C$$

In order that the lamps may not differ from one another by more than quite a small amount in brightness, it is necessary that the difference of P.D. between the nearest lamp C and the farthest E may not be more than 2 per cent., or say 2 volts on 100. Furthermore, it is obvious that this comparative uniformity between the nearest and farthest lamps will be quite unaffected by the loss along AC . Thus, it is general to fix the P.D. between the terminals of one lamp, viz. that nearest the dynamo, and cause the others to date from that.

Adding up all the losses between the lamps, it will be seen that the total loss is $r_2(3C + 2C + C)$, or $6r_2C$ volts, which is only half the loss which would have taken place had the whole current of $4C$ ampères been flowing all along the leads from C to E . From this follows a rule for ascertaining the loss along leads having a uniformly distributed load.

The loss of pressure in volts along leads carrying current to lamps uniformly distributed along their length is equivalent to the product of the whole resistance of the leads and the average current.

The above is the loss along that part of the leads bounded by the nearest and farthest lamp, and is independent of the

loss along the part AC (Fig. 4), which is equal to r_1 times the whole current. Thus, calling R_1 and R_2 the resistances respectively of the parts of the leads in between the dynamo terminals and the nearest lamp and in between the nearest and furthest lamps, and C the total current, then the loss from the dynamo to the nearest lamp is—

$$R_1 C$$

and the loss along the leads between the lamps is—

$$R_2 \frac{C}{2}$$

Uniformity of brightness in the lamps dictates that the latter should not be more than, say, 2 volts on 100; but this condition will not be affected by the loss $R_1 C$, provided the middle or some other convenient lamp be maintained at a certain agreed upon P.D., irrespective of the number of lamps in circuit. Economy of cost is the only consideration which will determine the loss along R_1 , and this will be considered later on.

It is usual to distinguish between these two parts of a circuit by calling that part of a system of leads which carries the same current throughout its whole length and is not tapped anywhere, a *feeder*; whilst that part of a system which is tapped at intervals along its length, and consequently carries a current which is not the same in all parts, is called a *distributor*.

A circuit may consist of a number of distributors connected together, and is then called a distributing network. Each portion of such network will have to be supplied by its own feeder to maintain the required P.D.

Fig. 5 will render this clear, where the double line of conductors bounding the figure represents the distributing network, which is connected to the station, or electricity works, S , by the feeders Sa , Sb , etc. Each point— a , b , c , etc.—is called a feeding centre, and is kept at a fixed P.D. by manipulation of the feeders in the station. The current for



the lamps situated in between a and b will be partly supplied by the feeder Sa and partly by Sb . The total current taken off the distributors between a and b being called C ampères, and being the same as that taken off the distributors between any two other feeding centres, it follows that each feeder will have to carry C ampères; and if R_f be the resistance of a feeder, lead and return, the loss of pressure in the feeder will

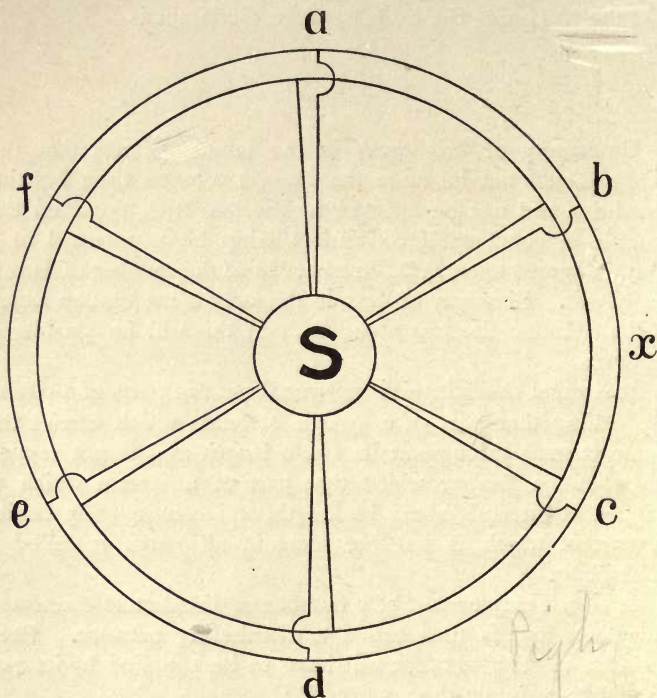


FIG. 5.

be CR_f , an amount which is not necessarily the same for all feeders, as their lengths may be different, or they may not be at all times equally loaded.

As the load between b and c (Fig. 5) is equally divided, the

current in the distributors will be $\frac{C}{2}$ ampères at the feeding centres, but gradually falling till some point is reached about the middle of the distance bc , say at x , where the current will be zero. In order, then, to state the maximum difference of P.D. between any two lamps, it is necessary to allow that the distributor in between a and b has a mean current of $\frac{C}{4}$ ampères, and a length equal to half the distance bc , say bx . Calling R_d the resistance of distributor ab , lead and return, the maximum difference of P.D. between a lamp at b and a lamp at x will be—

$$\text{Loss of volts along distributors} = \frac{C}{4} \times \frac{R_d}{2} = \frac{CR_d}{8} \text{ volts}$$

The determination of the size of the distributors depends upon the difference of P.D. allowable between any two lamps, and has already been considered. But in determining the size of the feeders other considerations are involved, such as economy of cost of transmission. If the feeders be of very small sectional area, they will cost little for material, but will waste much in heating, and the whole yearly cost made up of interest on cost of cables and laying, and value of energy wasted may be very large. On the other hand, if the feeder be of large sectional area it will cost much, whilst the energy wasted may be very small, but the total yearly cost may again be great. It is therefore necessary to take some intermediate size of leads which shall have the yearly cost as small as possible. This was first pointed out by Sir William Thomson (now Lord Kelvin), whose results may be expressed in the following simple rule—

The size of the feeders must be such that the interest for one year on money lying idle in their cost and laying together with depreciation, must be equal to the value of the energy wasted in them during one year.

Professor Bailly has pointed out a simple way of ascertaining that size of cable which gives the desired equality. Construct

two curves of cost (per mile, say) of cable for one year, thus—

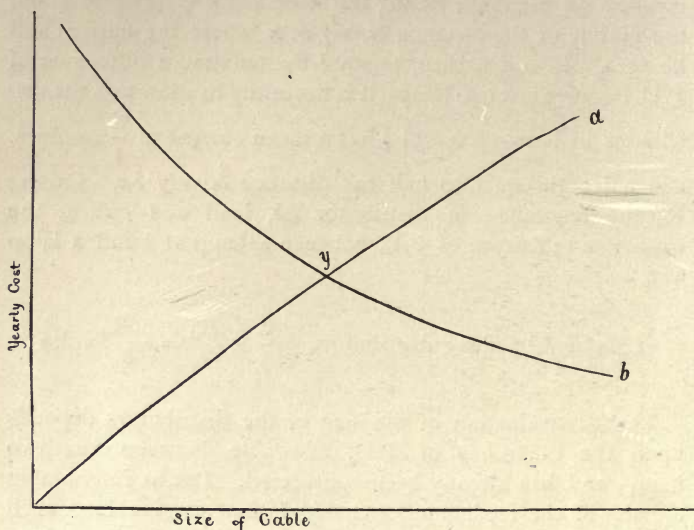


FIG. 6.

The curve *b* represents the cost of waste energy in the cable, whilst *a* represents the interest, etc., on the cost of the cable. The point *y*, where these two curves cut, shows where the two items are equal and also the size of cable.

In calculating out the loss in the feeders it is necessary to allow that no central station is on full load for all the twenty-four hours. It thus comes about that the current taken in the above calculation of waste in the cables must not be the full current; it should, in fact, be more nearly the—

$$\sqrt{\text{mean square}}$$

value when this can be ascertained.

A little consideration will at once show that strict adherence to Kelvin's law is not always desirable, seeing that where power is very cheap and materials and cost of laying very great, the most economical size of conductor might be such

that the energy waste was excessive, and though of no consequence as energy, yet producing a dangerous rise of temperature on the conductors.

A good many safety regulations limit the rise in temperature of the conductors to 150° Fahr. as a maximum, thus permitting a possible difference between the air and the conductor of about 75° Fahr., and that for a current twice as large as the normal maximum. Since the heating is, other things being equal, proportional to the square of the current, it follows that the rise in temperature due to the maximum working current must not be more than one-fourth part of the 75° Fahr., or about 18° Fahr. From a large number of experiments on conductors of all kinds, Mr. Kennelley has discovered a rule connecting the diameter of the conductor and the current to be carried consistent with the fact that the rise in temperature shall be within this limit, viz.—

$$C = 560\sqrt{d^3}, \text{ or } d = \frac{\sqrt[3]{C^2}}{68}$$

thus indicating that the current density should decrease as the diameter increases. As, however, most Fire Office rules fix the current density at 1000 ampères per square inch, irrespective of the size of the conductor, it follows that though for small sizes there is a very high degree of safety, nevertheless for conductors exceeding $\frac{1}{8}$ sq. inch sectional area the allowable current would produce a greater rise in temperature than that allowed by Kennelley's rule. Thus, when the sectional area is $\frac{1}{10}$ sq. inch, Kennelley's rule would allow about 17 per cent. more current for safety than the Phoenix Fire Office; when the area is $\frac{1}{8}$ sq. inch, the Fire Office rule would be equivalent to the safe temperature rise; whilst at 1 sq. inch area, the Fire Office rules allow 50 per cent. more current than is consistent with the maximum allowable rise in temperature.

Example XII.—The sectional area of the copper conductors forming a system of distributors is $\frac{1}{4}$ sq. inch, and they are loaded with one 50-watt 100-volt lamp per yard

of street. What must be the distance between the feeding centres, which are maintained at a P.D. of 100 volts, in order that the maximum difference between any two lamps shall not exceed 2 volts?

Solution.—Neglecting the small difference in the total current made by the fact that all lamps are not run at exactly the same P.D., viz. 100 volts, each lamp takes 0.5 ampère, and consequently the total current supplied by any feeder will be $\frac{L}{2}$ ampères, where L is the distance in yards between the two feeding centres. The resistance of distributor, lead and return, in between the feeding centres will be $2L \times$ resistance per yard, or—

$$2L \times 36 \times 4 \times 0.00000067 = 0.00019296L \text{ ohm}$$

and the loss of volts along half of L will be—

$$\frac{\frac{L}{2} \times 0.00019296L}{8} = 2$$

$$\text{whence } L^2 = \frac{16}{0.00009648} = 165,803$$

$$\text{and } L = 407 \text{ yards}$$

Example XIII.—What will be the maximum difference in volts between two lamps if the distance between two feeding centres is a quarter of a mile, the size of distributors is $\frac{1}{4}$ sq. inch sectional area, and current is taken off at the rate of $\frac{1}{8}$ ampère per foot of street?

Solution.—The maximum current in distributors is—

$$\frac{1}{8} \times \frac{1}{2} \times 13,200 = 110 \text{ ampères}$$

and therefore the average value is 55 ampères. The resistance of $\frac{1}{4}$ mile (lead and return) of distributor of $\frac{1}{4}$ sq. inch sectional area is—

$$4 \times 1320 \times 12 \times 0.00000067 = 0.04244 \text{ ohm}$$

whence the loss of volts is $0.04244 \times 55 = 2.334$ volts, which is consequently the maximum difference between two lamps.

Multiple Wire Systems.—In some cases the distributors are more than two in number, three or even five conductors being used. This method was first proposed by Dr. John Hopkinson. The connections are made as shown in Fig. 7, taking the five-wire system as an example—

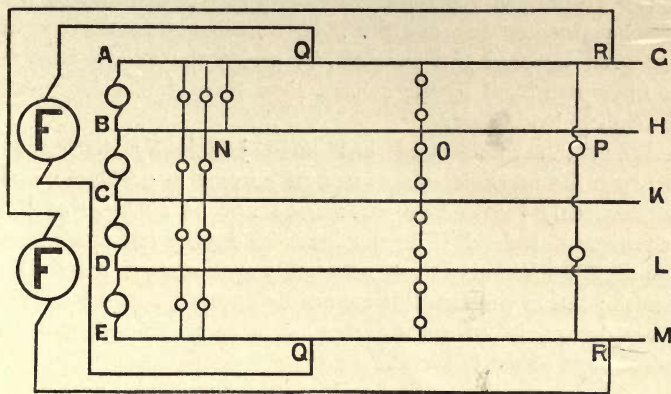


FIG. 7.

The five wires constituting the distributors are shown at AG, BH, CK, DL, EM, the outermost conductors AC and EM being considerably larger in sectional area than the three others. The object of the system is to gain the advantages of transmission of energy from the central station at a high pressure, say 400 volts, and nevertheless provide for the use of 100-volt lamps.

A three-wire system can do the same, using 200-volt lamps, which are now coming in. To transmit the same amount of energy at an E.M.F. of 400 volts, will only require one-sixteenth the weight of copper for the same percentage loss as on a 100-volt circuit. Lamps may be connected in various ways, as at N, 100-volt lamps in parallel between the conductors; as at O, two arc lamps being connected in series between each pair of distributors; and as at P, where 200-volt lamps are connected between the middle and outside wires.

The objects of the middle wires BH, CK, and DL, are—

(1) To enable the equality of P.D. between the wires to be maintained. This is done by introducing between each of them in the station a dynamo or other device, maintained at P.D., of 100 volts each.

(2) To prevent any want of equality in the numbers of the lamps connected between the parallels causing fluctuations in the P.D. between the parallels, by giving the excess current another path back to the station than through the lamps on the lightly loaded side.

When the whole load has been judiciously distributed between the parallels this excess of current is not large, and consequently the middle wires need not be so large as the outermost. Indeed, if all the loads be equally balanced, then the middle wires may be of so small a section as to practically vanish; but in practice this cannot be ensured, and the middle wires are made of such a size as to carry an agreed-upon percentage of the full load.

Like the simple parallel system, the distributors are fed by feeders connected to large dynamos running at a P.D. of 400 volts, plus the economical loss of volts in the feeders, conveying the current to the distributors at feeding centres as shown at QQ and RR.

Example XIV.—Compare two systems. No. 1. A two-wire system to supply current to a circle of distributors 4700 yards round, at the rate of one 50-watt 100-volt lamp per foot of street, and having ten feeding centres at equal distances round the circle, the station being at an average distance of 800 yards from the distributors. There is not to be more than 2 volts difference between any two lamps, and the pressure at the station end of the feeders may be 10 per cent. more than at the distributors.

No. 2. The same town to be supplied on the five-wire system with the same conditions; provision, of course, to be made for the connection of the distributors to the potential equalizers in the station.

Compare the weights of copper used in the two cases.

Solution.—No. 1. The distance between the feeding centres is 470 yards, whence the average current in the distributors is—

$$\frac{470}{2} \times 1.5 \times \frac{1}{2} = 176 \text{ ampères, say}$$

and thus the resistance of the distributor must be—

$$\frac{2 \times 2}{176} = 0.02273 \text{ ohm}$$

for a 2-volt drop. And as the length of distributor from a feeding centre to feeding centre is (lead and return)—

$$2 \times 470 \times 36 = 33,840 \text{ inches}$$

the resistance per one inch is—

$$\frac{0.02270}{33840} = 0.0000006712 \text{ ohm}$$

whence the sectional area is 1 sq. inch.

The feeders have to carry a current of $4 \times 176 = 704$, say 705 ampères, and are 800 yards long, with 10 volts loss. Thus the resistance is—

$$\frac{10}{705} = 0.01418 \text{ ohm}$$

or—

$$\frac{0.01418}{800 \times 2 \times 36} = 0.000000246 \text{ ohm per 1 inch}$$

and the sectional area is—

$$\frac{671}{246} = 2.728 \text{ sq. inches}$$

Taking 1 cubic inch of copper as 0.32 lb., the total weight of distributors will be—

$$\frac{4700 \times 36 \times 0.32 \times 2}{2240} = 48.34 \text{ tons}$$

and the feeders—

$$\frac{10 \times 800 \times 36 \times 2 \times 2.728 \times 0.32}{2240} = 224.4 \text{ tons}$$

Or the whole weight of copper in No. 1 = 272.74 tons.

In No. 2, as the outside wires will be at 400 volts P.D., the current will only have an average value of $\frac{1}{4}$ as much as in No. 1, or 44 ampères. Also the fall of P.D. may now be 8 volts, being only 2 per cent. as before. If the feeding centres are at the same distance apart, the sectional area of the distributors will thus be only $\frac{1}{16}$ that of No. 1, or $\frac{1}{16}$ sq. inch. But their length will be increased by 2×800 yards to connect them to the potential equalizers in the station; or the whole weight of outside wires will be—

$$\frac{(4700 + 800) \times 2 \times 36 \times 0.32 \times \frac{1}{16}}{2240} = 3.53 \text{ tons}$$

and allowing $\frac{1}{3}$ the section for the three middle wires each, their combined weights will be half as much as the outsides, or 1.76 ton.

The feeders also carry $\frac{1}{4}$ the current, and with four times the loss of volts they will likewise be $\frac{1}{16}$ the area of those in No. 1, or weight will be $\frac{272.74}{16} = 14.04$ tons, or the total weight will be 19.33 tons, as against 27.274 tons for No. 1.

CHAPTER III.

CIRCUITS CONTAINING MORE THAN ONE E.M.F.

IN some cases there is more than one E.M.F. in the circuit, and the consideration of the magnitude of the phenomena set up will depend upon whether all the E.M.F.'s are in one direction, or whether some are opposed to the others. The cases where all the E.M.F.'s are in one direction have already been dealt with, and we will now deal with some cases where the effective or active E.M.F. is the difference between an applied E.M.F. and another E.M.F., due to the nature of the circuit, acting against the applied E.M.F., and consequently called Back E.M.F.

Let E = an E.M.F. applied to a circuit to (1) overcome the B.E.M.F., and (2) leave sufficient over to send the necessary current through the resistance of the circuit.

ϵ = the B.E.M.F., such as that possessed by chemical cells, running motors, arc lamps, etc.

e = the active E.M.F., which is equivalent to the difference between E and ϵ , or $= E - \epsilon$, and which is also equal to the product of the resistance of the circuit and the current passing.

Thus $E = e + \epsilon$, or $e = E - \epsilon$, and consequently the current, which will be produced by E in a circuit of resistance R and containing the B.E.M.F. ϵ , will be—

$$C = \frac{E - \epsilon}{R} = \frac{e}{R}$$

The resistance R may be composed of various items as before.

Secondary Batteries.—In the case of charging a Secondary Battery, the circuit must consist of a dynamo, the Secondary Battery, and the necessary connecting cables. Each of these items will have their own resistances, that of the Secondary Battery being composed of a number of items in series, each being the resistance of a single cell. In addition to this, each cell will have an E.M.F., which must be overcome in charging, and this E.M.F. is a quantity which changes with the state of charge, being small when the charge is low, and rising to a higher value as the charge increases. This rise of E.M.F. is partly due to actual chemical change, and partly due to a variable temporary increase of local resistance at the surface of the plates. It is, however, customary to refer to it on the whole as an increase of the B.E.M.F. The curve in Fig. 8 shows the values of this E.M.F., corresponding with different charges.

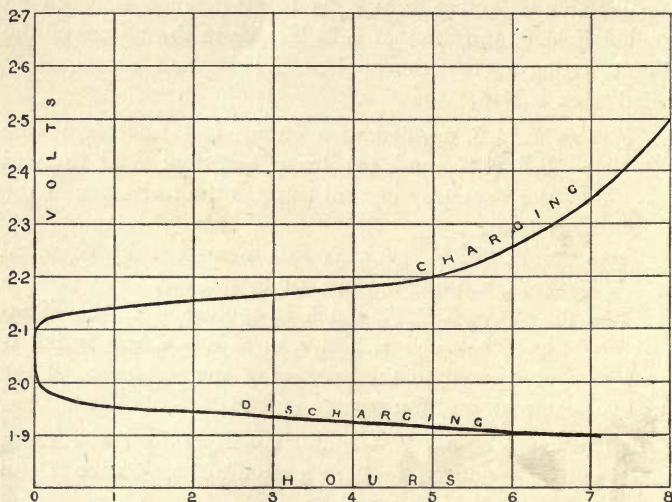


FIG. 8.

From this it will be observed that, though a cell may require 2.3 volts or so to overcome its B.E.M.F. when charging, it

cannot give more than a fraction more than 2 volts for even a short time when discharging.

We may thus be required to state what E.M.F. a dynamo must produce in order to charge so many secondary cells with such and such a current; or we may have to find what will be the value of the current through the cells when charged throughout with a constant E.M.F.

Example XV.—What must be the E.M.F. to charge with 100 ampères a battery of 54 cells at an average B.E.M.F. of 2·3 volts each, and each having an internal resistance of 0·0004 ohm, when the resistance of the dynamo is 0·02 ohm and that of the leads is 0·03 ohm?

Solution.—The total B.E.M.F. is—

$$\epsilon = 54 \times 2\cdot3 = 124\cdot2 \text{ volts}$$

and the active E.M.F. required for the resistance of the circuit will be the product of current into the sum of all resistances, or—

$$\begin{aligned} e &= 100(0\cdot02 + 0\cdot03 + 54 \times 0\cdot0004) \\ &= 100(0\cdot0716) = 7\cdot16 \text{ volts} \end{aligned}$$

whence the applied E.M.F. must be—

$$E = 124\cdot2 + 7\cdot16 = 131\cdot36 \text{ volts}$$

Example XVI.—What will be the current through the circuit at the beginning and end of charging in the case of a battery of 53 cells, each of 0·0002 ohm internal resistance, charged by a dynamo at an E.M.F. of 135 volts constant, when the dynamo resistance is 0·015 ohm, and that of the leads is 0·025 ohm, if the charging is continued for 7 hours, the value of ϵ per cell rising from 2·1 volts to 2·35 volts?

Solution.—The B.E.M.F. at starting to charge will be $2\cdot1 \times 53 = 111\cdot3$ volts, and at the end of 7 hours will be $2\cdot35 \times 53 = 124\cdot55$ volts. The total resistance is—

$$53 \times 0\cdot0002 + 0\cdot015 + 0\cdot025 = 0\cdot0506 \text{ ohm}$$

the active E.M.F. at the start of charging will thus be—

$$e = E - \epsilon = 135 - 111\cdot3 = 23\cdot7 \text{ volts}$$

and at the end of charging—

$$e = 135 - 124.55 = 10.45 \text{ volts}$$

whence the current at starting will be—

$$c = \frac{23.7}{0.0506} = 468 \text{ ampères}$$

and at end of run is—

$$c = \frac{10.45}{0.0506} = 206 \text{ ampères}$$

Example XVII.—In the last example (XVI.), if the dynamo has to be run so as to produce 135 volts always, and it is desired that the current shall not exceed 200 ampères, what will be the value of the necessary resistance to be inserted (1) at starting, (2) at end of charging?

Solution.—The active E.M.F. at starting—

$$e = 23.7 \text{ volts}$$

and at end of run is—

$$e = 10.45 \text{ volts}$$

whence the total resistance at starting must be—

$$\frac{23.7}{200} = 0.1185 \text{ ohm}$$

and at end of run—

$$\frac{10.45}{200} = 0.05225 \text{ ohm}$$

But the resistance of the circuit is already 0.0506 ohm, whence the added resistance must be—

$$0.1185 - 0.0506 = 0.0679 \text{ ohm at start}$$

$$\text{and } 0.05225 - 0.0506 = 0.00165 \text{ ohm at end of charging.}$$

Example XVIII.—A battery of 55 cells has a discharging E.M.F. of 2.05 volts per cell at start, and gradually falling to 1.9 volts each cell at end of 7 hours' run. The internal resistance of each cell is 0.0004 ohm. The resistance of the leads from the cells to the lamps is 0.005 ohm. What number of

cells must be used to supply 400·50-watt 100-volt lamps if the P.D. at the lamp terminals is to be kept at as nearly 100 volts as possible all the time?

Solution.—The lamps are to take 100 volts, and will also require $\frac{50}{100} \times 400 = 200$ ampères, neglecting the slight difference made by the P.D. at their terminals not always being exactly 100 volts.

The loss of volts along the leads is—

$$0\cdot005 \times 200 = 1 \text{ volt}$$

and the loss in the cells is—

$$0\cdot0004 \times 200 = 0\cdot08 \text{ volt per cell}$$

or the available E.M.F. per cell is $2\cdot05 - 0\cdot08 = 1\cdot97$ volt at start, or—

$$1\cdot90 - 0\cdot08 = 1\cdot82 \text{ volt at end of run}$$

The E.M.F. required for leads and lamps is to be—

$$100 + 1 = 101 \text{ volts}$$

whence the number of cells required will be—

$$\frac{101}{1\cdot87} = 51\cdot2, \text{ say } 51 \text{ at start}$$

and—

$$\frac{101}{1\cdot82} = 55\cdot4, \text{ or } 55 \text{ cells at end}$$

Motors.—In running motors we are also dealing with circuits containing a B.E.M.F. A motor is a dynamo caused to run by sending through its armature a current at sufficient E.M.F. to overcome the B.E.M.F., and leave sufficient to maintain that current through the resistance of the circuit. We have, then, as before, that the current in the armature will be—

$$C = \frac{E - \epsilon}{R}$$

R being the resistance of the whole circuit, and ϵ the B.E.M.F.

produced by the rotating armature. The part of the energy which is supplied to the motor that is converted into mechanical work is $C\epsilon$, and of this a part is expended in friction of various kinds in the motor itself; this part will be dealt with later on, and at present will simply be expressed as a waste in watts or horse-power. The difference between $C\epsilon$ and this waste is the available power of the motor, and is usually expressed as so many brake horse-power (B.H.P.).

Example XIX.—A motor having a resistance of $0\cdot025$ ohm, and producing a B.E.M.F. of 90 volts at a given speed, is connected by leads of $0\cdot005$ ohm resistance to a dynamo of resistance $= 0\cdot02$ ohm. It is required to produce a B.H.P. of 7 H.P. What must be the E.M.F. of the dynamo if the waste in internal friction, etc., in the motor is 300 watts?

Solution.—The total mechanical power is—

$$C\epsilon = C \times 90 \text{ watts}$$

of which 300 watts are wasted in the motor, leaving 7×746 watts available at the brake. Therefore—

$$C \times 90 - 300 = 7 \times 746$$

$$\begin{aligned} C &= \frac{5522}{90} \\ &= 61\cdot35 \text{ ampères} \end{aligned}$$

And this current is produced through the resistance of the whole circuit by an active E.M.F.—

$$e = C (\text{resistance of circuit})$$

or—

$$\begin{aligned} e &= 61\cdot35(0\cdot025 + 0\cdot005 + 0\cdot02) \\ &= 3\cdot0675 \text{ volts} \end{aligned}$$

whence the total E.M.F. must be—

$$90 + 3\cdot0675 = 93\cdot0675 \text{ volts}$$

Example XX.—In the above question, what will be the brake horse-power of the motor if the total E.M.F. of the

dynamo is 95 volts, assuming the same speed and loss due to friction, etc.?

Solution.—Here the active E.M.F. will be—

$$e = 95 - 90 = 5 \text{ volts}$$

and consequently the current will be—

$$C = \frac{5}{0.05} = 100 \text{ ampères}$$

therefore the total mechanical power is—

$$100 \times 90$$

and of this 300 watts is still the waste, leaving for brake power—

$$9000 - 300 = 8700 \text{ watts}$$

or—

$$\frac{8700}{746} = 11.6 \text{ H.P. at brake}$$

Arc Lamps.—In the case of arc lamps, the energy required for the vaporization of the carbons is generally considered as made up of the product of the current and an E.M.F. at the crater, called a back E.M.F., the magnitude of which is about 39 volts for a continuous current arc; but which appears to be as low as 30 volts (virtual) for some alternating current arcs. Thus, in an arc lamp, part of the E.M.F. supplied is concerned in sending the current through the resistance of the circuit, and the remainder is to overcome the back E.M.F.

Example XXI.—An arc lamp has a B.E.M.F. of 39 volts, and is connected to leads at P.D. of 50 volts. The resistance of the lamp leads is 0.11 ohm, the resistance of the lamp coil is 0.09 ohm, and the carbons have a resistance of 0.08 and 0.12 ohm respectively, whilst the arc itself has a resistance of 0.1 ohm. What must be the resistance included in the circuit so that the lamp may take a current of 10 ampères?

Solution.—The available or active E.M.F. is—

$$e = 50 - 39 = 11 \text{ volts}$$

and since the current is to be 10 ampères, we have the total resistance must be—

$$\frac{e}{C} = \frac{11}{10} = 1.1 \text{ ohm}$$

But the resistance of the lamp itself is—

$$0.11 + 0.09 + 0.08 + 0.12 + 0.1 = 0.5 \text{ ohm}$$

whence the additional resistance required must be—

$$1.1 - 0.5 = 0.6 \text{ ohm}$$

E = 4.74 \times 10^8 \phi N n w

CHAPTER IV.

DYNAMOS AND MOTORS.

IN questions relating to the proportions of parts of dynamos and motors we have to consider the relationship between the armature and magnet windings, etc. The electro-motive force generated in a revolving armature can be arrived at from the data of its parts. Thus, the whole generated E.M.F. is—

$$E = \frac{Nnw}{10^8}$$

for a Bipolar machine, whether drum or gramme wound, where—

N = whole number of c.g.s. magnetic lines which traverse the armature core,

n = the revolutions per second of the armature,

and w = the whole number of wires counting all round the circumference of the armature,

10^8 = the number of c.g.s. units of E.M.F., equivalent to the practical unit, the volt.

The factor Nn stands for the rate of cutting lines in c.g.s. lines per second.

The number of lines N is made up of the product of the sectional area of the iron armature core and the number of c.g.s. lines per unit of sectional area. It is very usual to call the number of c.g.s. lines per square centimetre the value of β , or—

β = c.g.s. lines per sq. cm. of armature iron

For different types of armature there are different values of β most suitable. Thus, for gramme or cylinder armatures, β

may range from 14,000 to 18,000 sq. cms., or even higher ; whilst, owing to the necessity of passing the lines through the same size of air gap in the drum as in the cylinder type, we must keep the value of β for drums at about 9000 to 12,000 per sq. cm., though in some types of drum higher values can be used with advantage.

The speed n is limited by the strength of the structure, on the one hand, to not more than 3000 to 4000 feet per minute (roughly, 1500 to 2000 cms. per second) circumferential speed ; or, on the other hand, to far smaller values where slow speeds are desirable for direct coupling, to slow-speed engines, for example.

The number of wires all round, w , is determined by the E.M.F. desired, and their size by the current to be carried. As the commutator connects the armature to the outside circuit in two parallels in Bipolar machines, the size of wire on the armature is that to carry half the current from the machine. No fixed rule can be stated for the relationship between the current and the size of wire, as this will depend upon the size of machine, the efficiency desired, and the rise in temperature allowable. This part of the subject is considered in a later chapter.

The whole E.M.F. E is the total volts generated by the armature, and, of course, a part of this is required to circulate the current in armature itself. Thus, if—

r_a = resistance of armature winding measured from brush to brush,

and C = total current in ampères in the armature, then Cr_a volts will be lost in the armature, and the difference $E - Cr_a$ is the terminal P.D. of the dynamo, in the case of a magneto machine or a shunt dynamo.

The resistance r_a is obviously the resistance of $\frac{1}{2}$ the whole length of wire upon it (since the two halves, and in parallel, and each half is half the resistance of the whole), plus any contact resistance at the brushes and in the leads to the brushes.

Leaving for the present the exact statement of the relationship between the armature and magnet windings, we will take a few simple examples.

Example XXII.—What will be the sectional area of iron required in the armatures respectively of two dynamos, one ring wound and the other drum wound, each having 240 wires all round, running at 1200 revolutions per minute, and giving a total E.M.F. each of 120 volts?

Solution.—In each case we have that the number of lines N required will be—

$$N = \frac{E 10^8}{nw}$$

and since $n = \frac{1200}{60} = 20$ revolutions per second

$$N = \frac{120 \times 10^8}{240 \times 20} = 2,500,000 \text{ lines total}$$

Now, the ring-wound armature may have $\beta =$ say 15,000, whilst the drum may have β only 10,000, say.

$$\text{But } N = \beta a$$

whence in the ring type—

$$a = \frac{2,500,000}{15,000} = 166\frac{2}{3} \text{ sq. cms.}$$

and in the drum—

$$a = \frac{2,500,000}{10,000} = 250 \text{ sq. cms.}$$

area of iron core.

Example XXIII.—What is the total E.M.F. produced by a dynamo, having an armature ring or gramme wound with 204 wires all round, when $\beta = 16,000$, area of cross-section of iron core is 125 sq. cms., and speed is 25 revolutions per second?

Solution.—

$$E = \frac{Nnw}{10^8} = \frac{16,000 \times 125 \times 25 \times 204}{10^8} = 102 \text{ volts}$$

Example XXIV.—At what speed must the above armature be run so as to give 102 volts terminal pressure when the resistance of the armature is 0.02 ohm, the current is 100 ampères, and the value of β remains at 16,000?

Solution.—The loss of volts in armature resistance will be—

$$Cr_a = 100 \times 0.02 = 2 \text{ volts}$$

whence the total generated E.M.F. must be—

$$E = 102 + 2 = 104 \text{ volts}$$

thus the speed—

$$n = \frac{E10^8}{Nw} = \frac{104 \times 10^8}{16,000 \times 125 \times 204} = 25.49 \text{ revolutions per sec.}$$

or 1529.4 per minute.

Example XXV.—What is the value of β in an armature core, drum wound, 25 cms. diameter, with 7 cm. hole in the centre, and composed of 500 discs half a millimetre thick, if wound with 180 wires all round, and run at 1200 revolutions per minute, producing an E.M.F. of 175 volts at the terminals, when the resistance of the winding is 0.032 ohm, and the current is 100 ampères?

Solution.—The total generated E.M.F. is—

$$\begin{aligned} E &= 175 + Cr_a \\ &= 175 + 100 \times 0.032 \\ &= 178.2 \end{aligned}$$

and therefore—

$$N = \frac{178.2 \times 10^8}{20 \times 180} = 4,950,000$$

But $N = \beta \times \text{area}$, and area is—

$$a = (25 - 7) \times 500 \times 0.05 = 450 \text{ sq. cms.}$$

and therefore—

$$\beta = \frac{4,950,000}{450} = 11,000 \text{ per sq. cm.}$$

Example XXVI.—An armature, gramme wound, has a core 36 cms. diameter, with 23 cm. hole, and is built up of 1000 discs, each 0.28 millimetre thick. There are 192 wires all round, each large enough to carry a current of 60 ampères. The speed is 781.2 revolutions per minute, and the resistance of the armature is 0.039 ohm. What is the value of β required to produce a terminal P.D. of 150 volts at a current of 120 ampères?

Solution.—At 120 ampères the loss of volts in the armature is—

$$Cr_a = 120 \times 0.039 = 4.68 \text{ volts}$$

whence the generated E.M.F.—

$$E = 150 + 4.68 = 154.68 \text{ volts}$$

thus to produce this E.M.F.—

$$N = \frac{154.68 \times 10^8}{\frac{781.2}{60} \times 192} = 6,188,000 \text{ lines}$$

and the sectional area of the iron core is—

$$a = (36 - 23) \times 1000 \times 0.028 = 364 \text{ sq. cms.}$$

whence—

$$\beta = \frac{N}{a} = \frac{6,188,000}{364} = 17,000 \text{ per sq. cm.}$$

Magnetization of Field-magnets.—So far we have been considering cases where the exact value of the current in the armature is given. In the majority of cases, however, the armature has to carry a current which is greater than that in the external circuit by the amount taken by the coils to magnetize the field-magnets. This magnetization is brought about in several ways, viz.—

1. By current from some independent source — called separate excitation, thus—

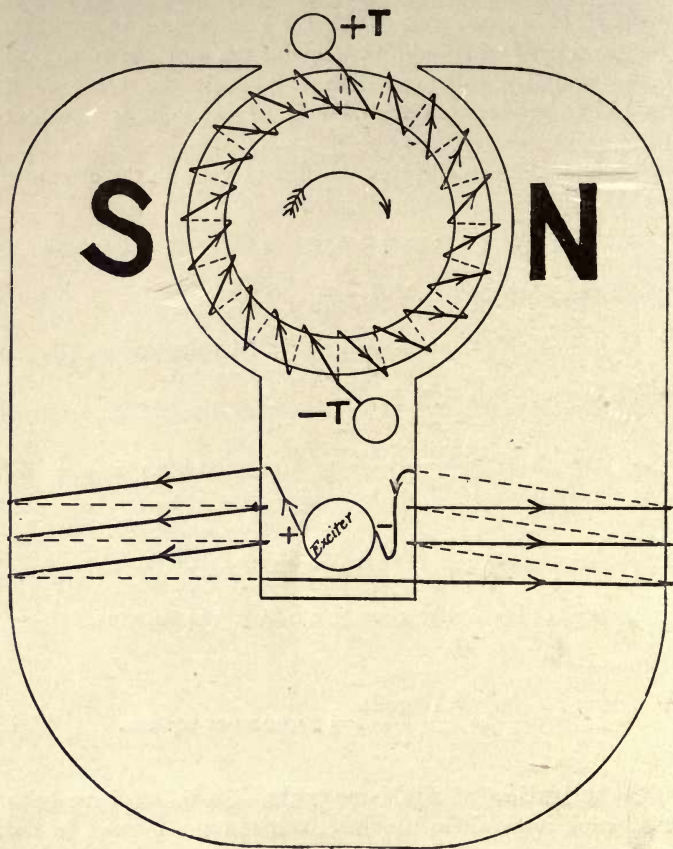


FIG. 9.

Here the data of the magnet-winding are of no particular importance in the present connection.

The current in the armature is the same as the current in the external circuit.

2. By a current from the armature of the machine itself

caused to circulate round the magnet-winding before going to the outside circuit, thus—

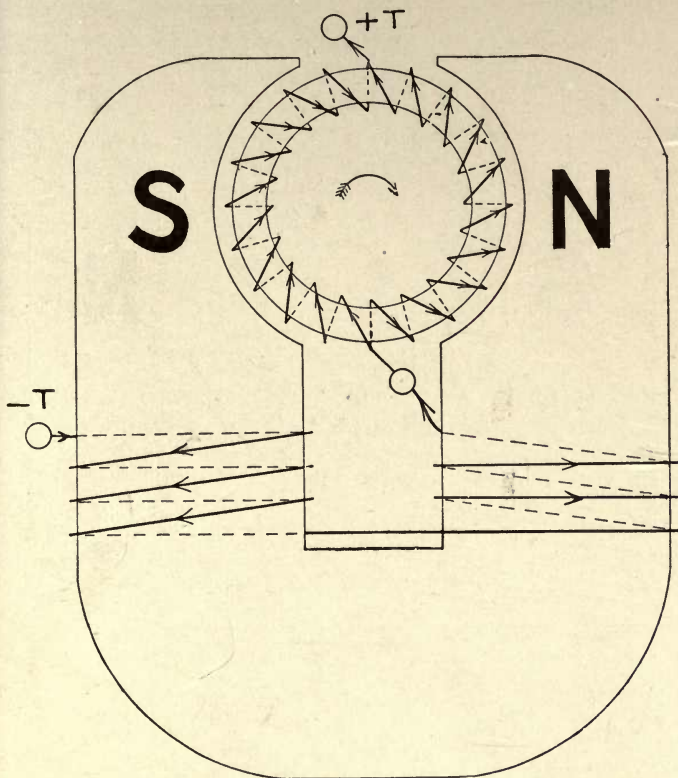


FIG. 10.

where $+T$ and $-T$ represent the terminals of the machine which are to be connected to the external circuit.

This is called *series* winding; and here there is a loss of volts in the resistance of the series coil, which has to be added to the loss in the armature to get the terminal P.D.

In this case also the current in the armature is the same as the current in the outside circuit.

We will call r_m the resistance of the series coil.

3. Sometimes the magnet legs are excited by a current derived from the terminals of the machine, called a *shunt* winding, thus—

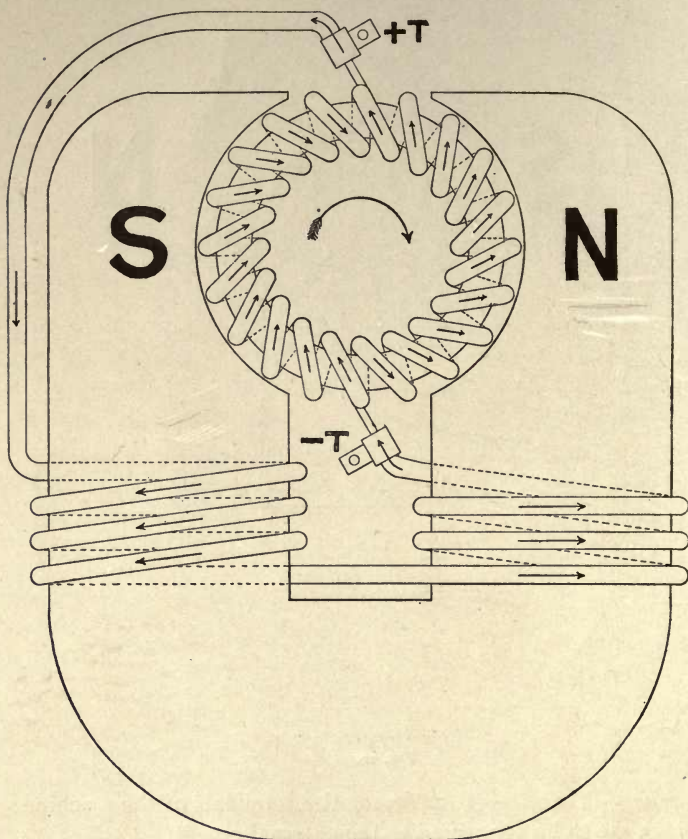


FIG. II.

In this case it is clear that the armature has in it a current which is greater than that in the external circuit, by the amount taken by the shunt coil. The total resistance of the circuit is composed of the armature in series with the external circuit

which is in parallel with the shunt coil. Calling r_s = the resistance of shunt coil, and r_a that of the armature, and the current in the external circuit = C ampères due to a terminal P.D. of e watts, we have that the current in the armature—

$$C_a = C + \frac{e}{r_s}$$

and the loss of volts in the armature will be—

$$C_a r_a = r_a \left(C + \frac{e}{r_s} \right)$$

whence the total generated E.M.F.—

$$E = e + r_a \left(C + \frac{e}{r_s} \right)$$

(4) There may, for certain reasons, to be considered later on, be a combination of shunt and series excitation, and it is possible to connect up these two coils in two different ways; firstly, thus—

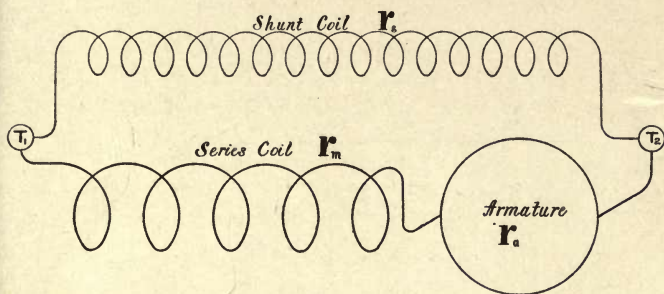


FIG. 12.

called “long shunt.” Compound wound.

In this case, calling e = terminal P.D. ; C = current in the external circuit, etc., as before, we have that the current in the shunt—

$$C_s = \frac{e}{r_s}$$

and the current in the series coil and armature will be both the same, viz.—

$$C_a = C + \frac{e}{r_s}$$

whence there will be a loss of volts in the series coil of—

$$r_m \left(C + \frac{e}{r_s} \right) \text{ volts}$$

and in the armature of—

$$r_a \left(C + \frac{e}{r_s} \right) \text{ volts}$$

The total loss in the machine will be—

$$(r_a + r_m) \left(C + \frac{e}{r_s} \right) \text{ volts}$$

and thus the total generated E.M.F.—

$$E = e + (r_a + r_m) \left(C + \frac{e}{r_s} \right)$$

Or, secondly, connections may be made thus—

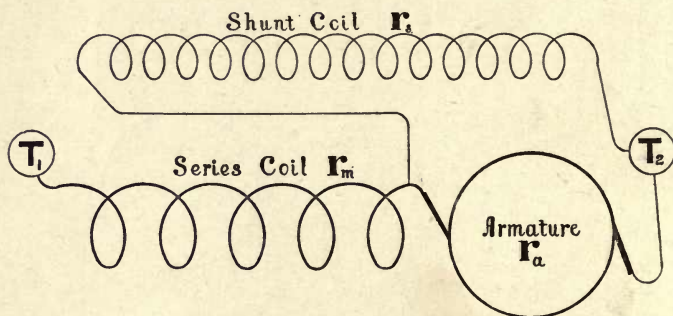


FIG. 13.

which is called "short shunt." Compound wound.

Here there will be a loss of volts in the series coil, due to the main current C , or this loss will be—

$$Cr_m \text{ volts}$$

whilst the current in the armature is greater than C , the external current, by the amount taken by the shunt coil.

Calling e the terminal P.D. as before, we have that the P.D. at the terminals of the shunt coil will be—

$$e + Cr_m$$

and thus the current in the armature will be—

$$C_a = C + \frac{e + Cr_m}{r_s}$$

and the loss of volts in the armature is thus—

$$r_a \left(C + \frac{e + Cr_m}{r_s} \right)$$

whence the whole E.M.F.—

$$E = e + Cr_m + r_a \left(C + \frac{e + Cr_m}{r_s} \right)$$

There are other important differences between “long” and “short” shunt winding, which will have to be considered later on.

Example XXVII.—A shunt dynamo is to give a current of 50 ampères at a terminal P.D. of 104 volts. The resistance of the armature is 0.04 ohm, that of the shunt coil 52 ohms. There are 210 wires all round, and the sectional area of the core is 160 sq. cms. The speed is 1080 revolutions per minute. If the winding is gramme type, what will be the value of β ? ✓

Solution.—The current in the armature—

$$C_a = C + \frac{e}{r_s} = 50 + \frac{104}{52} = 52 \text{ ampères}$$

whence the loss of volts in the armature is—

$$52 \times 0.04 = 2.08 \text{ volts}$$

or the total generated E.M.F.—

$$E = 104 + 2.08 = 106.08 \text{ volts}$$

to produce which E.M.F. the total number of lines—

$$N = \frac{106.08 \times 10^8}{\frac{1080}{60} \times 210} = \frac{10,608,000,000}{0.18 \times 210}$$

$$= 2,806,000 \text{ lines}$$

whence the value of the magnetic density is—

$$\beta = \frac{2,806,000}{160} = 17,530 \text{ per sq. cm.}$$

Example XXVIII.—At what speed must a series machine run to give a current of 10 ampères at a terminal P.D. of 1000 volts, if the resistance of the armature is 2 ohms, and that of the magnet coils 1.8 ohm, when the area of cross-section of the armature core is 150 sq. cms., and there are 17,500 c.g.s. lines per sq. cm.? The winding is ring type, and there are 2100 wires all round.

Solution.—The loss of volts in the armature is—

$$10 \times 2 = 20 \text{ volts, and in the magnet coil is—}$$

$$10 \times 1.8 = 18 \text{ volts}$$

or a total loss of 38 volts; whence the generated E.M.F.—

$$E = 1000 + 38 = 1038 \text{ volts}$$

The total number of lines through the armature is—

$$N = 150 \times 17,500 = 2,625,000 \text{ lines}$$

whence the speed—

$$n = \frac{1038 \times 108}{2100 \times 2,625,000} = 18.83 \text{ revolutions per second}$$

or 1129.8 per minute.

Example XXIX.—A long shunt compound dynamo is run so as to light 200 50-watt 100-volt lamps at the end of leads of resistance of 0.008 ohm. The resistance of the armature is 0.022 ohm, that of the series coil 0.015 ohm, and that of the shunt is 60 ohms. If the speed be 1200 per minute, what will be the sectional area of iron core required when $\beta = 17,085$, and there are 102 wires all round the armature?

Solution.—The current in the lamps is—

$$C = 100 \text{ ampères}$$

And there is a loss of 100×0.008 , or 0.8 volts in the leads; whence the terminal P.D. is 100.8 volts; and thus the shunt current is—

$$C_s = \frac{100.8}{60} = 1.68 \text{ ampère}$$

and consequently the current in the series coil and armature will be—

$$C_a = 101.68 \text{ ampères}$$

the loss of volts in the armature and series coil will be—

$$101.68 \times (r_a + r_m)$$

or—

$$101.68 \times (0.022 + 0.015) = 3.76 \text{ volts}$$

then for the whole generated E.M.F.—

$$E = 100.8 + 3.76 = 104.56$$

to produce which the number of lines total will be—

$$N = \frac{10,456,000,000}{20 \times 102} = 5,125,490$$

and as $\beta = 17,085$, the sectional area—

$$a = \frac{5,125,490}{17,085} = 300 \text{ sq. cms.}$$

Example XXX.—A short shunt compound dynamo is required to give a current of 200 ampères through 53 secondary cells of 0.00025 ohm each internal resistance, and having a B.E.M.F. when charging of 2.41 volts each cell. The leads to the cells are 0.005 ohm resistance; the series coil is 0.008 ohm; the armature is 0.012 ohm; and the shunt resistance is 50 ohms. At what speed must she run when there are 144 wires all round the armature, the sectional area of the armature core is 1100 sq. cms., and there are 17,100 c.g.s. lines per sq. cm.?

Solution.—The total B.E.M.F. of the cells is—

$$\epsilon = 2.41 \times 53 = 127.73 \text{ volts}$$

there is a loss in their internal resistance of—

$$200 \times 53 \times 0.00025 = 2.65 \text{ volts}$$

the loss in the leads is—

$$200 \times 0.005 = 1 \text{ volt}$$

and that in the series coil is—

$$200 \times 0.008 = 1.6 \text{ volt}$$

Thus the P.D. at the terminals of the shunt coil is—

$$127.73 + 2.65 + 1 + 1.6 = 132.98, \text{ say } 133 \text{ volts}$$

and due to which the current in the shunt coil is—

$$C_s = \frac{133}{0.50} = 2.66 \text{ ampères}$$

Whence the current in the armature will be—

$$C_a = 202.66 \text{ ampères}$$

and the loss in the armature—

$$202.66 \times 0.012 = 2.43 \text{ volts}$$

and thus the total generated E.M.F. is—

$$E = 133 + 2.43 = 135.43 \text{ volts}$$

to produce which the speed must be—

$$n = \frac{13,543,000,000}{1100 \times 17,100 \times 144} = 5 \text{ revolutions per second}$$

or 300 per minute.

Pull on Conductors.—The relationship between the power required to drive an armature as a dynamo and the current and E.M.F. produced, is that the E.M.F. depends upon the speed only, as is shown from the formula we have already used, viz.—

$$E = \frac{Nnw}{10^8}$$

since for any given machine N and w are fixtures. The current which the machine will give depends upon the resistance of the circuit connected to it, and upon the power which

the prime mover can put into it. There is a magnetic effect produced by the current in the armature windings, which acts so as to resist the motion. This will be understood by reference to Fig. 11.

The prime mover has to overcome the resistance to motion, called a drag on the winding, by exerting a pull on the one side of the belt attached to the dynamo. The magnitude of this pull can be simply expressed as—

$$P = CL\beta_g \times 0.0000002244 \text{ pounds per wire}$$

where C = the current in ampères in any wire,

L = the length of a wire passing through the field in cms.

β_g = the strength of the magnetic field in the air gap in c.g.s. lines per sq. cm.

Owing, however, to the want of uniformity in the strength of the magnetic field round the armature, the actual pull on any wire will depend upon its position in the air gap. When, for instance, the wire under consideration is midway in between the polar horns, the pull will practically be zero, since there the field is very small; but when under a pole piece the pull will be considerable, though gradually varying from a smaller to a larger value as the wire moves round the polar arc. The average value of the pull is that given above, and the total pull on the belt, as far as production of electricity is concerned, will be the product of this average pull and the number of wires all round which can be considered to be under the magnetic influence, or cutting lines. This number is approximately equal to—

$$w_1 = w \frac{2\theta}{360} \times 1.1 = 1.1 \frac{w\theta}{180}$$

for bipolar machines. Where w is the whole number of wires counted all round, θ is the angle of polar embrace, and 1.1 is a numeric to take in the stray field in between the poles.

The angle θ varies from 120° to 140° generally, though it is sometimes outside these limits.

Thus the total pull will be—

$$P = \frac{C}{2} L \beta_g w \frac{\theta}{180} \times 1.1 \times 0.0000002244$$

where C = the total current from the machine in ampères,

L = the length of any wire that is practically the length of the bore of the magnets,

β_g = density of lines c.g.s. per sq. cm. in the air gap,

w = total number of wires all round,

θ = the polar angle,

and the formula is only for a bipolar machine. Simplifying, we have—

$$P = CL\beta_g w \theta \times 0.000000006856$$

From the above, the rate of doing work can be found by considering that this pull is a weight of so many pounds lifted through a distance equivalent to the mean circumference of the armature once in the time taken by one revolution; thus let—

d = mean diameter in feet of the armature, *i.e.* diameter of core plus the depth of winding.

n = number of revolutions per second.

Then the above weight is lifted—

$$\pi d \text{ feet in } \frac{1}{60n} \text{ minutes}$$

or the work done in one minute is—

$$60Pn\pi d \text{ foot-pounds}$$

whence the power is—

$$\begin{aligned} & \frac{60Pn\pi d}{33,000} \text{ horse-power} \\ = & \frac{63 \times 22ndCL\beta_g w \theta \times 0.000000006856}{7 \times 33,000} \\ = & \frac{3917.7ndCL\beta_g w \theta}{10^{15}} \text{ horse-power} \end{aligned}$$

For a dynamo this is, of course, only that part of the power required which is concerned in the production of electricity;

the power to overcome frictional losses of various kinds must be added. But in the case of a motor the above gives the mechanical power developed, part, however, of which is also lost in friction, etc.

In the case of motors the E.M.F. produced by the rotation of the armature is opposed to the supplied E.M.F., and is called a back E.M.F. ϵ . But as far as the motor is concerned this B.E.M.F. is one of the factors of the power, the other being the current; this B.E.M.F. has to be calculated exactly as though we were dealing with a dynamo.

Example XXXI.—At what speed will the armature of a series machine run, and what will be the brake H.P. if a current of 20 ampères be supplied to it at an E.M.F. of 400 volts? Its armature has a resistance of 0·25 ohm; its magnet winding 0·15 ohm. The loss in friction is 380 watts. There are 900 wires all round, and the total magnetic flux through the armature is two million lines.

Solution.—Since the total loss of volts in the machine is—

$$20 \times (0\cdot25 + 0\cdot15) = 8 \text{ volts}$$

there will be—

$$400 - 8 = 392 \text{ volts}$$

available for overcoming the B.E.M.F. And the speed necessary to produce this will be—

$$n = \frac{\epsilon 10^8}{Nw} = \frac{392 \times 10^8}{900 \times 2 \times 10^6} \\ = 21\cdot77 \text{ revolutions per second}$$

Thus the total rate of doing work mechanically is—

| | | |
|----------------------|-------------------------------|--------------|
| | $392 \times 20 = 7840$ watts, | of which the |
| loss in friction is— | <u>380</u> watts, | leaving— |
| | 7460 watts, | or— |

10 H.P. available at the brake.

Example XXXII.—A machine run as a shunt dynamo, with a current in the armature of 100 ampères, gives 100 volts at its terminals at a speed of 1200 revolutions per minute. The resistance of the armature is 0·035 ohm. What will be

the speed if this machine be run as a motor off mains at 100 volts, and taking a current of 102 ampères total, when the resistance of the shunt coil is 50 ohms, and the loss due to friction, etc., is at the rate of 325 watts?

Solution.—As a motor the shunt coil takes 2 ampères, leaving 100 ampères for current through the armature. As a dynamo there was a loss in the armature of $100 \times 0.035 = 3.5$ volts, whence the total generated E.M.F. must have been 103.5 volts, at a speed of 1200 revolutions per minute. As a motor there will also be the same loss of 3.5 volts in the armature, leaving only 96.5 volts for overcoming B.E.M.F., which B.E.M.F. will be produced at a speed of—

$$\frac{96.5}{103.5} \times 1200 = 1118 \text{ per minute}$$

Since the strength of the magnetic field will be the same as before, also the total mechanical rate of doing work will be—

$$\begin{array}{ll} C_e = 100 \times 96.5 = 9650 \text{ watts,} & \text{of which the} \\ \text{loss in friction is—} & \underline{325 \text{ watts,}} \quad \text{leaving—} \\ & 9325 \text{ watts for external work, or—} \end{array}$$

$$\frac{9325}{746} = 12.5 \text{ H.P. at the pulley}$$

Example XXXIII.—A shunt machine running at 720 revolutions per minute as a dynamo, gives 50 volts P.D. at its terminals at a current of 200 ampères in the external circuit. Its armature resistance is 0.0147 ohm, and that of its shunt coil is 12.5 ohms. At what speed will it run as a motor, and what current will it take at a P.D. of 50 volts to develop 12 brake H.P., when $\frac{1}{2}$ H.P. is lost in friction in the gearing, and 0.352 H.P. in the machine?

Solution.—The total rate of doing work by the motor must be $12 + 0.5 + 0.352 = 12.852$ H.P., or $12.852 \times 746 = 9588$ watts, which must be the product of the current in the armature and the B.E.M.F. produced. The shunt coil takes—

$$\frac{50}{12.5} = 4 \text{ ampères}$$

and thus the total current will be $C = C_a + 4$.

As the resistance of the armature is $0\cdot0147$ ohm, the loss of volts therein will be $C_a \times 0\cdot0147$ volts, and thus—

$$\epsilon = 50 - 0\cdot0147C_a$$

and $C_a\epsilon$ must be $= 9588$, or—

$$(50 - C_a \times 0\cdot0147)C_a = 9588$$

or—

$$- 0\cdot0147C_a^2 + 50C_a - 9588 = 0$$

whence $C_a = 204$ ampères, at which current the loss in the armature is—

$$204 \times 0\cdot0147 = 3 \text{ volts}$$

$$\text{leaving } 50 - 3 = 47 \text{ volts for } \epsilon$$

The total rate of doing work mechanically is—

$$204 \times 47 = 9588 \text{ watts}$$

and the speed is—

$$\frac{47}{53} \times 720 = 628\cdot4 \text{ per minute}$$

Example XXXIV.—A 10-ampère series machine which gives 1000 volts total E.M.F. at a speed of 1000 revolutions per minute, is supplied with a current of 10 ampères to run as a motor, and has to lift a weight of 3 tons at the rate of 50 feet per minute. If there is a loss due to friction in the gearing of $\frac{1}{2}$ H.P., and $0\cdot32$ H.P. lost in the friction, etc., in the motor, what will be the speed and P.D. of supply mains required if the resistance of the armature is $3\cdot5$ ohms, and that of the magnet coils $2\cdot5$ ohms?

Solution.—To lift 3 tons 50 feet per minute is doing work at the rate of—

$$\frac{3 \times 2240 \times 50}{33,000} = 10\cdot18 \text{ H.P.}$$

and, adding the losses, the total H.P. will be—

$$10\cdot18 + 0\cdot5 + 0\cdot32 = 11 \text{ H.P.}$$

or in watts $= 8206$, which is done by a current of 10 ampères against a B.E.M.F. of—

$$\frac{8206}{10} = 820\cdot6 \text{ volts}$$

to produce which the speed will be—

$$\frac{820.6}{1000} \times 1000 = 820.6 \text{ revolutions per minute}$$

To send 10 ampères through the resistance of the machine will require a P.D. of—

$$10(r_a + r_m) = 10 \times 6 = 60 \text{ volts}$$

whence the P.D. at the motor terminals will have to be—

$$820.6 + 60 = 881 \text{ volts, say}$$

Example XXXV.—The length of one wire on the surface of an armature is 20 cms., the mean diameter is 21 cms., the speed is 1200 revolutions per minute, and the density of lines in the air gap is 2291 per sq. cm. The angle of polar embrace is 135° , and there are 360 wires all round. If this machine be used to give a total current of 50 ampères, what will be the H.P. required to drive it if $\frac{1}{2}$ H.P. is lost in various ways?

Solution.—Applying the formula on p. 50, we have—

$$\text{H.P.} = \frac{3917.7 \times \frac{1200}{60} \times \frac{21}{30.47} \times 50 \times 20 \times 360 \times 135 \times 2291}{10^{15}}$$

= 6.01 H.P., to which must be added $\frac{1}{2}$ H.P. as above for friction, etc., or total H.P. required = 6.51 H.P.

Example XXXVI.—What will be the drag on one wire of a dynamo armature which gives a current of 800 ampères, and has an average magnetic field of 3500 c.g.s. lines per sq. cm. of air gap? The length of the polar bore is 80 cms., and the machine is bipolar.

Solution.—Using the formula given on p. 49, we have pull is—

$$P = CL\beta_g \times 0.0000002244 \text{ pounds}$$

and as the current here is the current in each wire, it is consequently 400 ampères, and—

$$\begin{aligned} P &= 400 \times 80 \times 3500 \times 0.0000002244 \\ &= 25.1328 \text{ pounds} \end{aligned}$$

CHAPTER V.

ARMATURE WINDING.

IN considering the size of wire on a given armature to carry a given current, we have to allow a certain temperature rise, and consequently have to take into account all the sources of heating. These are mainly threefold, viz.—

- (a) The $C_a^2 R_a$ loss due to the effect of the current in the armature on the resistance of the armature ;
- (b) The H loss due to molecular friction in the iron core, brought about by the changing of the magnetic flux ;
- (c) The F loss due to Foucault or eddy currents in the core, and other moving parts of the armature.

The energy developed in the armature due to these effects has to be got rid of at some reasonable temperature, and it is customary to fix a limiting difference of temperature between the surface of the armature and the outside air.

In calculating this temperature difference it is of course necessary to bear in mind that some part of the heating is got rid of in other ways than by convection and radiation from the outside surface of the winding. Thus, part is lost by conduction through the spider to the spindle, the winding to the commutator. Experience shows that this part so conducted away may be about $\frac{1}{4}$ of the total loss.

Mr. Esson has shown that the surface of an armature revolving at a circumferential speed of 3000 feet per minute, can dissipate energy at the rate of 0.00444 watts per sq. cm. of its surface covered by winding for every 1° C. temperature difference between that surface and the air. The corresponding figure for a stationary surface, such as a field-magnet coil, is 0.00281 watts per sq. cm. per 1° C. temperature difference.

A combination of these figures will give the rate of loss of heat for any armature in terms of the circumferential speed. Thus the increase in rate of cooling due to the speed of movement through the air of 3000 feet per minute (1500 cms. per second) is—

$$0.00444 - 0.00281 = 0.00163 \text{ watts}$$

per sq. cm. per 1° C., or—

$$\frac{0.00163}{3000} = 0.000000543 \text{ watts}$$

per sq. cm. extra per 1° C. difference of temperature per 1 foot per minute of speed through the air; or the loss for any speed may be put down as—

$$0.00281 + 0.000000543f \text{ watts}$$

per sq. cm. per 1° C., where f is the circumferential speed in feet per minute.

For a speed of $f = 3000$ feet per minute, we have that the total difference in temperature between the air and the armature surface will be—

$$T = \frac{W}{A \times 0.0044} \text{ degrees Centigrade}$$

where W = that part (say, $\frac{3}{4}$) of the whole expenditure of energy in the armature, which is considered to be radiated from its surface, expressed in watts;

and A = the surface area in sq. cms. of the armature.

Similarly, for a magnet coil we have—

$$T = \frac{W}{A \times 0.00281}$$

where A is the outside surface of the magnet winding, not including the cheeks of the bobbin, expressed in sq. cms.

The loss W is approximately $\frac{3}{4}(C_a^2 R_a + H + F)$, and of these items the first is, of course, quite easily found, allowance being made that, the temperature being high, we have to employ a high value for ρ .

The hysteresis loss H is due to the fact that molecular friction in the iron causes heating when the iron core as a

mass is rotated by the rotation of the armature, whilst the molecules of iron are themselves stationary. The effect produced is the same as if the molecules were rotated whilst the armature core remained stationary. In a bipolar field the armature core has all the magnetized molecules thus rotated once per revolution, and the value of the energy so expended has been found by Professor Ewing for all conditions of magnetization. The table on p. 214 gives these values expressed in ergs per cc. per cycle of change, one cycle corresponding to one revolution of an armature in a bipolar field. Now, as one watt-second is 10^7 ergs, we have that the total ergs expended in the armature per revolution multiplied by the speed in revolutions per second and divided by 10^7 will give the rate of loss in hysteresis in watts. Calling the value of the ergs per cc. per cycle h , we have that the total ergs per revolution will be hV , when V is the bulk of the armature iron in cc. If, then, n be the revolutions per second, we have the hysteresis loss is—

$$H = \frac{hVn}{10^7} \text{ watts}$$

The other loss, viz. that due to eddy currents, F , is almost impossible of calculation in an armature. Only a small part of it is in the laminated iron core, the major part being in other conducting masses, such as the winding, end plates, spider and spindle, etc. Due precautions being taken to minimize it as much as possible, we shall assume in these pages that it is equivalent to that part of the total loss which is got rid of in other ways than by radiation directly from the surface of the winding.

Therefore we may now put the value of W as—

$$W = C_a^2 R_a + H$$

and write the temperature rise as being—

$$T = \frac{C_a^2 R_a + H}{A \times 0.00444} \text{ degrees Centigrade}$$

for a bipolar armature revolving at 3000 feet per minute circumferential speed.



In finding the area of the surface of the armature, we must only take that part of the surface which is exposed to the air. In the case of a drum armature this is easy, but in a ring or gramme armature, some care is required in calculating the true size of that part of the surface which is inside the cylinder. In all cases the surface should be considered as smooth, no allowance being made for corrugations due to the curved surface of the wires; for the coefficients taken are stated higher to allow for such irregularity.

Example XXXVII.—What will be the hysteresis loss in an armature ring wound, 25 cms. long, 25 cms. diameter, with a 15-cm. hole (all dimensions being those of solid iron)? The speed is 1226 revolutions per minute, in a field of 4 million c.g.s. lines the value of the ergs per cc. per cycle is 10,750 per cc. for this particular iron.

Solution.—The sectional area of the armature is—

$$(25 - 15) \times 25 = 250 \text{ sq. cms.}$$

whence the density of lines is—

$$\beta = \frac{4 \times 10^6}{250} = 16,000 \text{ per sq. cm.}$$

corresponding to which the ergs per cc. per cycle are given as 10,750. The number of cycles per second will be the revolutions per second, or—

$$n = \frac{1226}{60} = 20.43$$

And the number of cc. of iron in the core will be—

$$V = 5 \times 25 \times 20 \times \pi = 7855 \text{ cc.}$$

whence we have that—

$$H = \frac{7855 \times 10750 \times 20.43}{10^7} = 172.51$$

say, 173 watts.

Example XXXVIII.—If the resistance of the armature in the last example is 0.0212 ohm when hot, and it is used to

light 200 50-watt 100-volt lamps at a terminal pressure of 102 volts, the resistance of the shunt coil being 51 ohms, what will be the rise in temperature due to hysteresis and $C_a^2 r_a$ losses, if the eddy current loss is got rid of by conduction, etc.? The surface of the armature is 3675 sq. cms., and the circumferential speed is to be taken as 3000 feet per minute.

Solution.—The current taken by the lamps is—

$$\frac{200 \times 50}{100} = 100 \text{ ampères}$$

and that taken by the shunt coil is 2 ampères, whence the total current in the armature is—

$$C_a = 102 \text{ ampères}$$

and thus—

$$C_a^2 r_a = (102)^2 - 0.0212 = 220.5, \text{ say } 221 \text{ watts}$$

and the loss due to H has been found to be 173 watts, whence the total watts—

$$W = 221 + 173 = 394 \text{ watts}$$

and thus the temperature rise will be—

$$T = \frac{394}{0.00444 \times 3675} = 24.1^\circ \text{ C.}$$

Example XXXIX.—Suppose the armature mentioned in Example XXXVII. were replaced by a drum having as before 25 cms. length of iron in it, but having only a 7-cm. hole, and run at a density of only 11,000 c.g.s. lines per sq. cm., corresponding to which the value of ergs per cc. per cycle is 5900—

(1) What will be the temperature rise with the same winding as to size of wire and the same current at the same E.M.F., and what the speed? and—

(2) What would be the temperature rise and output if the current and the speed are to be the same as when a gramme armature?

(3) Also, what would be the value of the current to make the temperature rise the same and the speed the same as in the case of the gramme?

The surface area of the drum may be taken as 4000 sq. cms., and the variations in the speed are not supposed to affect the rate of cooling.

Solution.—(1) As a drum there will be a sectional area of the armature of—

$$(25 - 7)25 = 450 \text{ sq. cms.}$$

whence at $\beta = 11,000$ the total number of lines will be—

$$N = \beta a = 11,000 \times 450 = 4,950,000$$

The total generated E.M.F. was before equal to 102 volts for the terminal P.D. plus the loss in the armature of—

$$102 \times 0.0212 = 2.16 \text{ volts, or } 104.16 \text{ total}$$

to produce which E.M.F. the drum will have to run at a speed inversely as the number of lines, or speed will be—

$$n = 20.43 \times \frac{400}{495} = 16.5 \text{ revolutions per second}$$

the volume of the drum core will be—

$$\frac{d_1^2 \pi l}{4} - \frac{d_2^2 \pi l}{4}$$

or—

$$V = \frac{25 \times 25 \times 22 \times 25}{4 \times 7} - \frac{7 \times 7 \times 22 \times 25}{4 \times 7} = 11,314 \text{ cc.}$$

Thus the hysteresis loss is—

$$H = \frac{5900 \times 11,314 \times 16.5}{10^7} = 110 \text{ watts}$$

And since in question (1) the current is to be the same as before, we have that the $C_a^2 r_a$ loss is 221 watts as before, whence the rise in temperature will be—

$$T = \frac{331}{4000 \times 0.00444} = 18.6^\circ \text{ C.}$$

(2) If the speed is to be the same as before, there will be a greater E.M.F. total in the ratio of—

$$\frac{495}{400}, \text{ or } \frac{495}{400} \times 104.16 = 128.9 \text{ volts}$$

whence with the same current the output will be—

$$100 \times 126.74 = 12,674 \text{ watts}$$

as against only 10,200 for the ring winding.

But at this speed of 20.43 revolutions per second the hysteresis loss will be—

$$H = \frac{5900 \times 11,314 \times 20.43}{10^7} = 136 \text{ watts}$$

or the total watts expended will be—

$$136 + 221 = 357 \text{ watts}$$

and the consequent temperature rise will be—

$$T = \frac{357}{4000 \times 0.00444} = 20^\circ \text{ C.}$$

(3) If, however, the temperature rise is to be the same as for the ring armature at the same speed, the current may be larger, such that the new $C_a^2 r_a$ loss added to the H loss will make the heating such as to cause a temperature rise of 24.1° C. Thus the total watts will be—

$$\begin{aligned} W &= T \times A \times 0.00444 \\ &= 24.1 \times 4000 \times 0.00444 \\ &= 428 \text{ watts} \end{aligned}$$

of which 136 watts are for hysteresis, leaving 292 watts for $C_a^2 r_a$. If the armature resistance be taken as 0.0212 ohm as before, then—

$$\begin{aligned} C_a^2 &= \frac{292}{0.0212} \\ \text{and } C_a &= \sqrt{\frac{292}{0.0212}} = 117 \text{ ampères} \end{aligned}$$

Example XL.—An armature working at a density of 8000 lines per sq. cm. and giving a certain E.M.F. is run at half the speed with a density of 16,000 per sq. cm. : what will be the ratio of the watts lost in hysteresis in the two cases?

Solution.—At the higher speed and $\beta = 8000$, h is 2860 (p. 215) ergs per cc. per cycle; and at the lower speed and $\beta = 16,000$, h is 10,400 ergs. But as the cycles per second are

only half as many in the second case as in the first, the two losses will be 2860 in the first case to $\frac{10400}{2} = 5200$ in the second case, or as—

$$\frac{2860}{5200} = 0.5499, \text{ say } 0.55$$

or the loss in H in the high speed low density machine is only 0.55 as great as in the slow speed high density.

Multipolar Machines.—When the dynamo is not bipolar, it may be so wound that either the current or E.M.F. is proportionally increased. Thus, with what is called the parallel winding, the coils are connected up as before to a commutator, which then has as many pairs of brushes as there are pairs of poles, unless the commutator is cross-connected, when all the positive brushes can be replaced by one brush, and similarly all the negative brushes by another brush. Using the same symbols as before, and N still standing for the number of lines entering the armature from or leaving the armature towards any one pole piece, the E.M.F. will be—

$$E = \frac{Nnw}{10^8} \text{ as before}$$

but the armature is now in $2p$ parallels instead of 2 parallels, where p is the number of pairs of poles, and can consequently carry p times as much current if the wire be the same size, or—

$$C_1 = pC$$

where C_1 is the current from the machine with p pairs of poles, and C the current from a bipolar machine with the same value for N , etc.

On the other hand, with the series winding, which is to be preferred to the parallel for various reasons, the E.M.F. is increased in proportion to the number of pairs of poles, whilst the current-carrying capacity of the armature is the same as before, being only in 2 parallels whatever the number of poles. Thus the E.M.F. will be—

$$E = \frac{Nnwp}{10^8} \text{ volts}$$

the current being simply the same as in a bipolar machine having the same size of wire.

The effect on the hysteresis loss in the core is the same in both cases and may be found by taking the product of the volume of the armature into h , n , and p and dividing by 10^7 —

$$\text{or } H = \frac{hVnp}{10^7} \text{ watts}$$

for both series or parallel windings.

Virtually the effect of the multiplicity of poles is the same as having p equal dynamos connected on the one hand in parallel and on the other in series.

Example XLI.—A six-pole dynamo has an armature core composed of 600 discs of iron 0.25 mm. thick, and having a radial depth of 7 cms. It is wound with wire which will carry 50 ampères, and has 540 wires all round. If the density of magnetic flux through the iron is 15,000 per sq. cm., and the speed is 6 revolutions per second, what will be the E.M.F. and current if connected up in the parallel method?

Solution.—The total value of lines is—

$$\begin{aligned} N &= 2 \times 600 \times 0.25 \times 0.1 \times 7 \times 15,000 \\ &= 3,150,000 \text{ lines} \end{aligned}$$

and the whole E.M.F. will be—

$$E = \frac{3,150,000 \times 6 \times 540}{10^8} = 102 \text{ volts}$$

As each wire will carry 50 ampères, the current, as an ordinary bipolar machine, would be 100 ampères; but there are now $2p = 6$ parallels in the armature, and thus the current is—

$$C = 6 \times 50 = 300 \text{ ampères}$$

Example XLII.—What will be the hysteresis loss in the above armature if the outside diameter of the core is 70 cms.?

Solution.—

$$H = \frac{hVnp}{10^7}$$

and h from the table on p. 216, is 9300. The volume of the core is—

$$V = 600 \times 0.025 \times 7 \times \frac{22}{7} \times 63 = 20,790 \text{ cc.}$$

and thus—

$$H = \frac{9300 \times 20,790 \times 6 \times 3}{10^7} = 348 \text{ watts}$$

CHAPTER VI.

THE MAGNETIC CIRCUIT.

THIS can best be illustrated and explained by a comparison with the electric circuit. The quantities corresponding to conductivity and resistivity, E.M.F., and current, are called, in the magnetic circuit, permeability and reluctivity, magnetomotive force (M.M.F.), and magnetic flux respectively. Permeability has this difference from conductivity, that it has no fixed value in the so-called magnetic materials (like the corresponding value of electric conductivity, which has a fixed value for a given temperature quite irrespective of the magnitude of the current or electric flux), but varies with the variation of the magnetic flux even when the temperature is kept constant.

In the non-magnetic materials, however, the value of the permeability is independent of the value of the magnetic flux, even when this is confined to the same path, and so varies in density. The use of the idea of current-density is not so usual as to have caused the adoption of a symbol for its indication; but for purposes of comparison we shall make use of D , as standing for the density in amperes per sq. cm. The corresponding symbol very frequently used in magnetic measure is β , and stands for the number of c.g.s. lines of magnetic flux per sq. cm. of cross-section.

It is very usual to confine β to the magnetic density in iron or the other magnetic metals, but for sake of uniformity we shall not so restrict its use, but let it stand for the density of magnetic lines in any material. It has been shown before

that we may write the relationship between the three quantities e , C , and R in the electric circuit in a simple way, viz.—

$$C = \frac{e}{R}, \text{ and } R = \frac{e}{C}$$

also—

$$R = \frac{l\rho}{a} = \frac{l}{am}$$

Adding now to our symbols the above-mentioned D , the ampères per sq. cm., we shall have that—

$$C = Da$$

and—

$$e = CR = DaR = Dl\rho = \frac{Dl}{m}$$

or, putting again C for the total electric flux—

$$e = \frac{Cl\rho}{a} = \frac{Cl}{am}$$

The corresponding quantities in the magnetic circuit we will call—

\mathfrak{M} = magneto-motive force (M.M.F.) measured in terms of a unit, to be afterwards defined.

$N = \beta a$ = the whole magnetic flux, generally represented in the form βa , on account of the importance of knowing the value of the density.

The quantity corresponding to resistance is the magnetic reluctance, and may be represented by \mathfrak{R} , whose magnitude will have to be stated in terms of the permeability, since there is no unit in use called unit relativity. Thus—

$$\mathfrak{R} = \frac{l}{a\mu}$$

where μ stands for permeability in the same way as we have used m to stand for conductivity, that is the actual conductance of the unit cube. We thus call μ the actual permeance of the unit cube, measured in terms of its value for some particular material, viz. air, in which μ is always regarded as unity.

Consider a solenoid of infinite length having a current of C ampères passing through it. Let the turns or convolutions of wire on it be w per cm. length. It is found that there will be produced inside the solenoid a magnetic flux having a density of $\frac{4\pi}{10}Cw$ lines per sq. cm. of cross-section of core, provided the core consist of air, or some other so-called non-magnetic material. This $\frac{4\pi}{10}Cw$ is commonly called H , and, owing to the permeability of air, etc., being unity, it follows that—

$$H = \frac{4\pi}{10}Cw = \text{the magnetizing force per unit length}$$

and it is in this sense that H is generally used. The magnetic materials are chiefly iron, steel, nickel, and cobalt, all others being practically non-magnetic, or having a permeability of unity; some few, such as bismuth, have a permeability slightly less than unity, but only to a quite insignificant extent. The so-called magnetic materials have a permeability much greater than unity, not constant, however, but depending upon the density of magnetization; that is, upon the magnitude of the magnetic flux through a given cross-section. We must also class a vacuum as non-magnetic, and having a value of $\mu = 1$. Thus, if the core of our infinite solenoid be replaced by a bar of iron, we have in the first place expelled all the air, but the ether may be regarded as being still present, and we now have the effect of two cores in parallel, the original core with $\mu = 1$, and a new core of iron with μ much greater than unity. The magnetizing force is still—

$$\frac{4\pi}{10}Cw \text{ per cm. length}$$

and is undisturbed, and consequently we get a magnetic flux of density H through the original ether core, and a magnetic flux through the new core of magnitude—

$$I = \frac{4\pi}{10}Cw \times \mu_1$$

when μ_1 is the value of the permeability of the iron core.

Adding these two magnetic fluxes together, we have—

$$\beta = I + H$$

the density of magnetic flux in the composite core.

From a purely physical point of view it is important to distinguish between I and H ; but as we can never actually separate them in practice, we invariably speak of β , their sum, and it is much more convenient to so consider the matter.

Though the idea of current density is sometimes used, we are not so familiar with it as we are with actual current total; but a little consideration will show that the magnitude of the E.M.F. required to produce a given current density D in a conductor depends simply upon the material and length of the conductor, or, more simply, for a given material of given length at a given temperature the E.M.F. required is proportional to the value of D , irrespective of the sectional area of conductor. Fortunately for the ease of calculation in the electric circuit, there is a very simple connection between the E.M.F. required and the value of D , viz. one of direct proportionality subject to the slight effect of temperature in altering the resistance of the conductor.

Thus the E.M.F. required for a given value of D was stated above as—

$$e = \frac{Dl}{m} \text{ or } = \frac{Cl}{am}$$

Similarly, calling the value of total magnetic flux $N = \beta a$, we have that the M.M.F. required is—

$$H = \frac{Nl}{a\mu} \text{ or } = \frac{\beta l}{\mu} \text{ in physical measure}$$

In the electric circuit the proportionality between e and D renders calculation easy without the use of tables; but in the magnetic circuit the want of direct proportionality between H and β renders the use of curves or tables absolutely necessary for exact work. The exact value of this relationship has been investigated by various observers, but we shall confine ourselves to the measurements made by Professor Ewing and the Drs. Hopkinson. A combination of their

results for good qualities of iron is shown in the tables given, which have been read off from carefully drawn curves, having a general appearance as shown in Fig. 14, which gives the relationship between—

$$\frac{4\pi}{10}Cw = H \text{ and } \beta$$

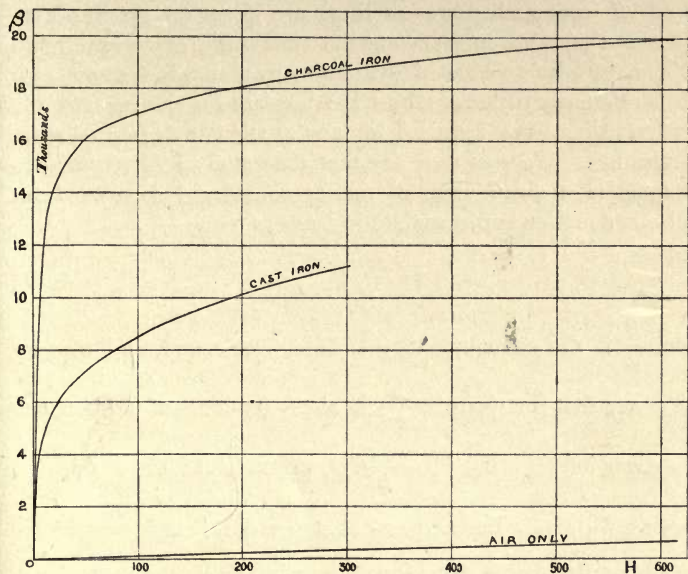


FIG. 14.

Ordinates stand for β , the c.g.s. lines per sq. cm. in the core of iron, and abscissæ for the magnetizing force $\frac{4\pi}{10}Cw = H$, required for every centimetre length of the iron. It will be noticed that when only feebly magnetized the iron has a much greater ratio of $\beta : H$ than when magnetized to a considerable extent, though at very low values of β this is not so noticeable. The three curves stand for this relationship for wrought iron, cast iron, and for air only. It has already been pointed

out that the iron adds to the value in air an amount called I , which is the difference between the ordinate of the air curve and that of the iron curve. This addition is varying in value, gradually increasing with the value of β , till, as the curve indicates, it is likely that at a very high value of H , this addition will have reached a maximum, and will then be simply a constant addition to the value for air alone. Tables carefully read off from such curves as these are given on pp. 214-218. It is not possible to represent the relationship between β and H for the whole extent of the curves by a simple formula, and thus reference to these tables is necessary for finding out the magnetizing force required for any particular degree of magnetization. We can thus say that the value of H required to magnetize a given piece of matter considered separate from all other matter and complete in itself as being—

$$H = \frac{IN}{a\mu} = \frac{l\beta}{\mu}$$

But $\frac{\beta}{\mu}$ is the same quantity as H in our curve, and thus we may say that the quantity $\frac{N}{a\mu}$ is some function of β expressed in terms of H . But $H = \frac{4\pi}{10}Cw$, which is the same thing as saying $\frac{4\pi}{10}$ times the ampère-turns per cm. length, and as we really apply magnetizing forces by winding with wire to carry a current, it is most convenient to do away with the numeric $\frac{4\pi}{10}$ by making allowance for it in the tables, thus—

$$H = \frac{4\pi}{10}Cw, \text{ or } Cw = H \frac{10}{4\pi}$$

$$Cw = H \times 0.795$$

or—

$$\text{ampère-turns per cm. length} = H \times 0.8 = f(\beta)$$

nearly enough, since our knowledge of the exact values of β and H is not so exact as to require a distinction between 0.795

and 0.8. The convenience of this was first pointed out by Mr. Esson, and has since been extensively made use of by engineers. Thus, in the tables, we have put in values of β with the corresponding values of H and $f(\beta)$, this last being the mode of writing *function of* β . The permeability μ is also shown, being readily found from—

$$\mu = \frac{\beta}{H}$$

These tables are for the magnetic circuit what tables, showing the relationship between D (ampères per sq. cm.) and e the P.D. required in volts per cm. length of conductor, would be for the electric circuit.

Using the symbols—

H = the whole magnetizing force required by a magnetic circuit, in physical measure called M.M.F.

we have—

The ampère-turns total = $0.8H = \mathfrak{M}$
 and the ampère-turns re- }
 quired by the part } = $0.8H = \omega$ = magnetic P.D.

Thus, remembering that it is just as necessary to have a complete circuit for the circulation of the magnetic flux as it is in the case of the electric flux, we have—

$$H = \frac{NI}{a\mu} = \frac{l\beta}{\mu} \text{ and } \mathfrak{M} = 0.8 \frac{l\beta}{\mu}$$

We must either have the whole circuit of one material of uniform quality and sectional area and of length l , in which case \mathfrak{M} is the whole magnetizing force required; or if l be the length of a part only of the circuit having a definite property, we must then regard—

$$0.8 \frac{NI}{a\mu} = \omega$$

the magnetic P.D. ampère-turns required by that part alone; whilst then the whole M.M.F. will be the sum of all such items as ω .

Thus in a composite magnetic circuit—

$$\mathfrak{M} = \sum 0.8 \frac{Nl}{a\mu}$$

$$0.8 \left\{ \frac{l_1 N}{a_1 \mu_1} + \frac{l_2 N}{a_2 \mu_2} + \frac{l_3 N}{a_3 \mu_3} \right\} \text{ etc.}$$

when the same total flux N circulates round the whole circuit.

The following comparison of the Electric and Magnetic Circuits will serve to sum up the previous considerations:—

| ELECTRIC CIRCUIT. | MAGNETIC CIRCUIT. |
|---|--|
| C = Current or whole electric flux in ampères. | N = Magnetic flux in c.g.s. lines total. |
| $D = \frac{C}{a}$ = ampères per sq. cm. of section, or current density. | $\beta = \frac{N}{a}$ = magnetic density or c.g.s. lines per sq. cm. |
| m = conductivity in mhos of the material of the circuit; it is the actual conductance in mhos of the unit cube. It is constant for a constant temperature irrespective of D . | μ = permeability of the material of the circuit, and is the actual permeance of the unit cube. It is constant only for air and the non-magnetic materials having a value of unity (1). It is variable for the magnetic materials, and dependent upon the value of β . |
| $\rho = \frac{1}{m}$ = the resistivity of the material or actual resistance in ohms of the unit cube. It is constant subject to the constancy of m . | $r = \frac{1}{\mu}$ = the reluctivity of the material or actual reluctance of the unit cube being constant, and of value unity (1) for air and the non-magnetic materials; but varies with μ in the magnetic materials, according to the value of β . |
| $R = \frac{l\rho}{a} = \frac{l}{am}$ = resistance in ohms of a circuit or part of a circuit of length l and area a made of material having resistivity ρ , | $\mathfrak{R} = \frac{lr}{a} = \frac{l}{a\mu}$ = reluctance of a circuit or part of a circuit of length l and area a made of material having a permeability μ , |

ELECTRIC CIRCUIT.

$$e = \frac{Cl}{am} = \frac{Dl}{m} = Dl\rho = \text{the D.P.}$$

in volts required to send a current of C ampères through a circuit or part of a circuit.

e can readily be calculated from any of the above relations.

E = the whole E.M.F. in the circuit, being the algebraic sum of all the P.D.s round the circuit, or the sum of all such quantities as—

$$D_1 l_1 \rho_1 + D_2 l_2 \rho_2 + D_3 l_3 \rho_3$$

Though these P.D.s may be required distributed round the circuit E may be generated at one part, viz. the armature of a dynamo, for example.

In the electric circuit it is quite easy to confine the electric flux to a very definite path, because some materials (air, for example) are practically of infinite resistivity. In the magnetic circuit, however, air and the other so-called non-magnetic materials have a permeability which is, at the very best, only $\frac{1}{3000}$ as great as that of iron, the very best magnetic conductor we have. There is thus no possibility of magnetically insulating any circuit to anything like the perfect extent attainable in the electric circuit; and we have consequently to allow for some leakage of lines. The permeability of iron varies

MAGNETIC CIRCUIT.

ω = The magnetic P.D. required to send a magnetic flux through a circuit or part of a circuit. It is equal

$$\text{to } 0.8N\mathfrak{A} = \frac{NI}{a\mu} 0.8.$$

ω can only be calculated for the non-magnetic materials where $\mu=1$. For the magnetic materials reference has to be made to the tables for the value of μ , etc.

By reference to the $J(\beta)$ column of the tables, the value of ω in ampère-turns per cm. length can readily be found, and calculation thereby much simplified.

\mathfrak{A} = the whole M.M.F., being the algebraic sum of all the magnetic P.D.'s round the circuit, or being equal to—

$$0.8 \times \left\{ \frac{\beta_1 l_1}{\mu_1} + \frac{\beta_2 l_2}{\mu_2} + \frac{\beta_3 l_3}{\mu_3} \right\}$$

Though these items may be required distributed round the whole circuit, the whole of \mathfrak{A} may be produced at one part, viz. the magnet coils of a dynamo, for example.

from about 3000 to 30 or so at such values of β as are convenient for general work, and thus the leakage of lines may often be as great even as if we had no better insulator than manganin, or such like, for the protection of the copper conducting path in the electric circuit. This leakage of lines is particularly the case in dynamos and such apparatus where the magnetic circuit is composite in character, and when the magnetic P.D. between point and point may be very great. Thus, if the permeance of the armature and air gaps in series be represented by u , and that of the surrounding air space be called w , these two permeances are in parallel, and are subjected to nearly the same magnetic P.D. Consequently, if N be the total magnetic lines through the armature core and air gaps, the magnetic P.D. between the polar faces will be $\frac{N}{u}$, and this

magnetic P.D. will produce a flux of $\frac{Nw}{u}$ lines through the surrounding air space. Now, these two fluxes have to flow in the magnet limbs, which will consequently carry, not N lines, but—

$$N + N \frac{w}{u} = N_m$$

In their paper on dynamos in 1886 the Drs. Hopkinson chose to represent this fact in the very simple way of stating the lines N_m in the magnet limbs as being ν times the lines in the armature core; the value of ν depending upon the relationship of w to u . Thus, when $w = u$, $\nu = 2$, a value met with in such multipolar machines as the old pattern of Ferranti alternator. The value of w is, however, generally much less than u in a well-proportioned dynamo, some measured values of ν being given in the following table :—

| Type of machine. | Value of ν . |
|---|------------------|
| Edison, Hopkinson, single magnet, drum ... | 1'32 |
| Siemens " " " ... | 1'30 |
| Phoenix over type " " gramme ... | 1'32 |
| Phoenix double magnet " ... | 1'40 |
| Manchester " " " ... | 1'49 |
| Victoria Mordey, 4 pole, ring or gramme ... | 1'40 |
| Ferranti, 16 pole, disc ... | 2'00 |
| Manchester with polar supports ... | 1'5 to 1'60 |

Professor Forbes, many years ago, showed how a sufficiently exact approximation to the value of ν for practical purposes could be calculated from the drawing of the machine.

Thus, the same magnetic P.D. which is required between the two polar faces to produce N lines in the armature and air gaps, is also producing a magnetic flux through the very imperfectly magnetically insulating air which surrounds the dynamo, and which additional magnetic flux has to be accommodated in the magnet limbs, thereby causing the value of β in them to be greater, and making the value of the magnetizing force for that part of the circuit greater than would otherwise be the case.

We have, then, to consider the magnetic circuit of the dynamo as composed of a number of items, each of which may be different, either on account of material or the fact that a greater flux of lines has to be accommodated. This composite magnetic circuit consists of—

(1) The armature, which is almost invariably made of a good quality of wrought iron. The sectional area is easily found, as has been indicated previously. The average length of the path of magnetic lines can be found with sufficient exactness by taking the mean semi-circumference in the case of a bipolar machine, whether drum or gramme; or for a multipolar machine it is—

mean circumference

$2p$

where p is the number of pairs of poles.

(2) The iron to iron space, called the air gaps, though mostly occupied by the winding, etc., on the armature. The sectional area of this part is equivalent to the area of the curved polar surface, increased by a fringe of $0.8 \times$ the iron to iron space. This increase is to take into consideration the spreading of the lines round the edges, and was first pointed out by the Drs. Hopkinson.

The length of this space is twice the iron to iron space.

(3) The magnet limbs, which may be wrought iron, cast iron, or cast steel. Their sectional area is easily found from the

drawing, and may vary in different parts, as, for instance, when the pole-pieces are of cast iron. The length is the same as that of a line drawn slightly nearer the centre of the machine than the centre of the limb, and bounded by the centre of the polar face on the one hand, and by the yoke on the other, taken twice over.

(4) The yoke, which is generally counted separately, either on account of difference of material or difference in sectional area. The area of section is easily found, whilst the length is that of a curved line joining the extremities of the lines representing the path through the magnet limbs.

Considerable experience and discretion is required to accurately fix these lengths.

Using the following symbols to represent these various measurements, viz.—

l_a , A_a , and μ_a for the armature, through which a total flux of N c.g.s. lines is passing, and in which, consequently, there

is a magnetic density of $\beta_a = \frac{N}{A_a}$

l_g , A_g for the iron to iron space or air gaps. In these the value of μ is 1, and consequently need not appear; and the magnetic flux is also N , as through the armature. The

density of lines in the gap is thus $\beta_g = \frac{N}{A_g}$

l_m , A_m , μ_m for the magnet limbs, through which a greater number of lines than N , viz. νN , will flow, and conse-

quently the magnetic density is $\beta_m = \frac{\nu N}{A_m}$

l_y , A_y , and μ_y for the yoke, in which again the total flux is νN ,

and the density $\beta_y = \frac{\nu N}{A_y}$

Thus the total M.M.F. in physical measure will be—

$$\mathfrak{M} = \frac{N l_a}{A_a \mu_a} + \frac{N l_g}{A_g} + \frac{\nu N l_m}{A_m \mu_m} + \frac{\nu N l_y}{A_y \mu_y}$$

And we have shown above that the ampère-turns can be found by multiplying the physical measure by 0.8, whence—

$$\left. \begin{array}{l} \text{Ampère-turns} \\ \text{total} \end{array} \right\} = 0.8 \frac{Nl_a}{A_a \mu_a} + 0.8 \frac{Nl_g}{A_g} + 0.8 \frac{Nl_m}{A_m \mu_m} + 0.8 \frac{Nl_y}{A_y \mu_y}$$

Or again, to save trouble in calculation, we can call the $0.8 \frac{N}{A \mu}$ the value of $f(\beta)$ in ampère-turns, and put—

$$\text{Ampère-turns total} = l_a f(\beta_a) + 0.8 l_g \beta_g + l_m f(\beta_m) + l_y f(\beta_y)$$

the 0.8 as a multiplier being retained in the case of the air gaps, since for air we have not made out a table or curve for reference, and consequently the 0.8 is not included there as it is in the $f(\beta)$ for iron.

Example XLIII.—How many ampère-turns will be required to produce a magnetic flux of 300,000 lines in a wrought-iron ring of 20 sq. cms. cross-sectional area, made of round bar iron, and being 50 cms. outside diameter?

Solution.—The value of—

$$\beta = \frac{300,000}{20} = 15,000$$

whence $f(\beta) = 22.88$ ampère-turns per cm. length.

The length of the magnetic circuit is, nearly enough, the mean circumference of the ring. From its sectional area of 20 sq. cms. the diameter of the bar is 5.046 cms., whence, nearly enough, the mean diameter is $50 - 5.046 = 45$ cms., say, and thus the circumference is—

$$45 \times \pi = 141.35 \text{ cms.}$$

whence—

$$lf(\beta) = 141.35 \times 22.88 = 3238 \text{ ampère-turns}$$

Example XLIV.—Neglecting the air path or leakage lines, what will be the magnetizing force required to produce 3 million lines through a mass of cast iron 1 metre long and 517 sq. cms. sectional area?

Solution.—The value of $\beta = \frac{3 \times 10^6}{517} = 5802$, and the magnetizing force in physical measure—

$$H = \frac{NI}{a\mu} = \frac{\beta l}{\mu}$$

corresponding to which value of β , $\mu = 290$, whence—

$$H = \frac{5800 \times 100}{230} = 2000$$

which is in physical measure, and will be converted into ampère-turns by multiplying by $\frac{10}{4\pi} = 0.8$, or—

$$\text{Ampère-turns} = 0.8 \times 2000 = 1600$$

Example XLV.—What will be the addition to the ampère-turns required for the iron ring in Example XLIII. if a saw slot, 1 mm. thick, be made through the iron bar in a plane containing the axis of the ring? Neglect leakage lines.

Solution.—There will be slightly less ampère-turns required for the iron, in as far as the iron is less in length by 1 mm. or 0.1 cm. As the magnetizing force is 22.88 per cm., we shall now require 2.288 less magnetizing force as far as the iron is concerned, viz. 2.288 ampère-turns less. But for the air gap, where the whole 300,000 may be taken as uniformly distributed over an area of 20 sq. cms., plus a fringe of 0.08 cm. all round, that is over an area equal to that of a circle $5.046 + 0.16 = 5.206$, say 5.2 cms. diameter, which has an area of 21.23 sq. cms.

Thus the density is—

$$\beta_g = \frac{300,000}{21.23} = 14,127 \text{ per sq. cm.}$$

and the ampère-turns required per cm. length will be $0.8 \times 14,127 = 11,301$. But as the gap is only 0.1 cm. long the additional ampère-turns will be only 1130.1 ampère-turns for air space.

Thus the difference will be an addition of—

$$1130.1 - 2.28 = 1127.82 \text{ ampère-turns}^*$$

* Strictly speaking, owing to leakage lines caused by the increased magnetic P.D. between the ends of the gap and points near to the gap, there will be leakage of lines outside the area of gap taken, and these lines will have to be accommodated in the iron part of the ring, thus making the ampère-turns for the iron greater than before.

Example XLVI.—What will be the ampère-turns required to magnetize a dynamo which has a ring armature 25 cms. diameter, with a 15-cm. hole, and built up of 1000 discs of iron 0·25 mm. thick, if the total magnetic flux through it is 4 million lines? The diameter of the bore is 27 cms., its length is 27 cms., and the angle of embrace is 135 degrees. The length of the two magnet legs together is 124 cms., with a sectional area of 415 sq. cms., that of the yoke being 54 cms. and 729 sq. cms. respectively. Both magnet legs and yoke are wrought iron, and the leakage coefficient $\nu = 1\cdot4$.

Solution.—Using symbols previously mentioned, we have—

$$A_a = (25 - 15)(1000 \times 0\cdot25 \times 0\cdot1) = 250 \text{ sq. cms.}$$

and also—

$$l_a = \frac{\pi}{2} \times \frac{25 + 15}{2} = 10\pi = 31\cdot5 \text{ cms. say}$$

thus—

$$\beta_a = \frac{4 \times 10^6}{250} = 16,000$$

whence the ampère-turns required for the armature will be—

$$\begin{aligned} m_a &= l_a f(\beta) = l_a f(16,000) \\ &= 31\cdot5 \times 39\cdot84 = 1254\cdot9 \end{aligned}$$

say 1255 ampère-turns for armature.

For the air gap we have—

$$l_g = 27 - 25 = 2 \text{ cms.}$$

and as the angle of embrace is 135° , the curved length of the pole-piece is—

$$\frac{135}{360} \times \pi \times 27 = 31\cdot8 \text{ cms.}$$

The length parallel with the shaft is 27 cms., and thus the area of the polar surface is—

$$31\cdot8 \times 27 = 858\cdot6 \text{ sq. cms.}$$

to which must be added the fringe all round of 0·8 times the iron to iron space, or in this case of 0·8 cm. all round, or—

$$0\cdot8(2 \times 27 + 2 \times 31\cdot8) = 94\cdot08 \text{ sq. cms.}$$

making—

$$A_g = 858.6 + 94 = 952.7, \text{ say, } 953 \text{ sq. cms.}$$

and thus—

$$\beta_g = \frac{4 \times 10^6}{953} = 4200 \text{ say}$$

whence the ampère-turns for gap = $0.8 l_g \beta_g$ —

$$\mathcal{M}_g = 0.8 \times 2 \times 4200 = 6720$$

Again, for the magnet legs we have the total number of lines—

$$N_v = 1.4 \times 4 \times 10^6 = 5.6 \times 10^6$$

and as $A_m = 415$, we have—

$$\beta_m = \frac{5.6 \times 10^6}{415} = 13,494$$

corresponding to which $f(\beta) = 10.9$ nearly enough, and the ampère-turns required will be—

$$\mathcal{M}_m = l_m f(\beta) = 124 \times 10.9 = 1352 \text{ ampère-turns say}$$

Lastly, for the yoke there will be the same total flux, 5.6×10^6 , whence—

$$\beta_y = \frac{5.6 \times 10^6}{729} = 7681$$

for which—

$$f(\beta) = 2.12$$

and therefore $\mathcal{M}_y = 54 \times 2.12 = 115$ ampère-turns say. Thus the whole magnetizing force will be in ampère-turns—

| | | | | |
|-------------|-----|-----|-----|-------------------------|
| Armature | ... | ... | ... | 1255 |
| Air gaps | ... | ... | ... | 6720 |
| Magnet legs | ... | ... | ... | 1352 |
| Yoke | ... | ... | ... | 115 |
| | | | | <hr/> |
| | | | | 9442 ampère-turns total |
| | | | | <hr/> |

Example XLVII.—The length of magnetic circuit of a transformer iron core is 50 cms. Its sectional area is 300 sq. cms., and the maximum value of the current allowed for magnetization is 0.15 ampère. How many turns of wire must

the primary coil have if the total magnetic flux is 1,200,000 lines?

Solution.—

$$\text{Density, } \beta = \frac{12 \times 10^5}{300} = 4000$$

for which $f(\beta)$ is 1.24, and therefore the ampère-turns required—

$$lf(\beta) = 50 \times 1.24 = 62$$

whence, the current being 0.15, the total turns must be—

$$w = \frac{62}{0.15} = 413 \text{ say}$$

CHAPTER VII.

MAGNET COIL WINDINGS.

WE have seen in the previous chapter how to ascertain the magnetizing force required in ampère-turns, and will now proceed to show how the size of wire and number of turns or convolutions may be found. There are two main cases—

- (1) That in which the current to be used is a fixture.
- (2) That in which the P.D. available is fixed.

The first is simple, and can be easily disposed of. Let \mathfrak{M} = the ampère-turns to be provided, and C the fixed current, then the number of turns is, of course—

$$w = \frac{\mathfrak{M}}{C}$$

The size of wire is then fixed, partly by the magnitude of the current to be carried, and partly by the fall of potential allowable. Generally this last is so limited that the expenditure of energy is so small as to make the heating negligible; but in any case the dimensions of the bobbin, or former, can be made such as to satisfy Mr. Esson's rule, given on p. 56.

The coils which mostly come under this, the "series," class, are those of ammeters, cut-outs, series-wound dynamos, series coils on compound dynamos, and independent coils generally.

In case (2), which may be called the "shunt" class, we have different conditions to take. A certain E.M.F., say E volts, is to set up such a current that this current, multiplied by the turns of wire, shall be equal to \mathfrak{M} , the ampère-turns required; or—

$$Cw = \mathfrak{M}$$

Since the whole resistance R will be—

$$R = wr$$

where r is the resistance of the mean turn, we have—

$$C = \frac{E}{wr}$$

and—

$$\mathfrak{M} = \frac{Ew}{wr} = \frac{E}{r}$$

or—

$$r = \frac{E}{\mathfrak{M}}$$

whence a simple rule :—

Shunt Coil.—*The resistance of the mean turn is equal to the P.D. at the terminals divided by the ampère-turns required; or is the ratio of the E.M.F. to M.M.F.*

It is easy in most cases to fix or find the length of the mean turn, say l , whence the sectional area of the wire will be—

$$a = \frac{l\rho}{r}$$

when ρ is the resistivity at the particular temperature to which the coil is to be raised. The diameter of the wire, if circular in section, is then —

$$d = \sqrt{\left(\frac{4a}{\pi}\right)} = \sqrt{\left(\frac{4l\rho}{r\pi}\right)}$$

Or, putting in the previously found value for the resistance of the mean turn, we have —

$$d = \sqrt{\left(\frac{4l\rho}{\pi \frac{E}{\mathfrak{M}}}\right)} = \sqrt{\left(\frac{4l\rho \mathfrak{M}}{\pi E}\right)}$$

when, of course, l and ρ must be in cms. or inches, according as d is wanted in cms. or inches. Also ρ should have the value corresponding to the temperature to which the coil is allowed to rise.

To find the number of turns, w , we must again have recourse

to the formula of Mr. Esson, and make the total expenditure of energy—

$$\frac{E}{R} \text{ or } \frac{E}{wr}$$

such as to make no greater rise of temperature than may have been previously agreed upon.

A little investigation will at once show that the number of turns can be varied considerably without altering the value of \mathfrak{M} when once the right size of wire has been found, provided the length of the mean turn be not thereby altered.

Thus with only one turn the energy waste will be—

$$\frac{E^2}{r}$$

and may be hopelessly large; but with a suitable number of turns, w , this waste will be reduced to—

$$\frac{E^2}{wr}$$

or to $\frac{1}{w}$ times its first value. Since, however, the ampère-turns are—

$$Cw \text{ and } C = \frac{E}{wr}$$

we have in both cases that—

$$\mathfrak{M} = \frac{E}{rw} \times w = \frac{E}{r}$$

whatever value w may have.

The number of turns it is possible to get on a coil is found when the size of the coil is known, and the depth to which it is possible to wind it. Thus let d_1 stand for the diameter of the covered wire (easily found for double cotton covering by adding 0.02" to the diameter of the bare wire in inches). Let L stand for the length of the bobbin or former which may be

occupied by wire, and D the depth to which the winding may be laid. Then there will be—

$$\frac{D}{d_1} \text{ layers}$$

each of—

$$\frac{L}{d} \text{ turns}$$

and—

$$w = \frac{LD}{d_1^2}$$

all quantities being, of course, expressed in the same units.

The value of \mathfrak{M} for dynamos, etc., is calculated as shown in a previous chapter; whilst \mathfrak{M} for ammeters, voltmeters, and other small instruments, is usually found in the first instance by experiment. Many ammeters and voltmeters of the magnetic type are alike in size of parts whether ammeter or voltmeter, being only different in the winding. Now, it is evident that, with all the different ranges of reading required, it is not possible to provide exactly any particular number of ampère-turns with any particular current or P.D. Thus all ammeters and voltmeters of this class have some power of adjustment that enables the nearest value of \mathfrak{M} to be made use of. In ammeters it is not always possible to make the product cw = the value of \mathfrak{M} desired, and owing to the trade range in sizes of wires for voltmeters, it is not always possible to find that size in stock that will make the right value of resistance of mean turn. Thus both ammeters and voltmeters are calculated for on the assumption that the value of \mathfrak{M} is variable within certain limits. In some instruments of the permanent magnet class \mathfrak{M} ranges round 300 ampère-turns, whilst in some of the electro-magnet type it may range round 600 ampère-turns for ammeters, and about 400 ampère-turns for voltmeters.

CHAPTER VIII.

ARMATURE REACTIONS.

So far we have only considered the simplest case of dynamo-winding, and will now proceed to discuss the calculations for different types of machines. In order to appreciate what it is we have to allow for in a dynamo at full load, we shall have to consider the effect of the current in the armature and the magnetic circuit. In Fig. 15 is shown a section through a

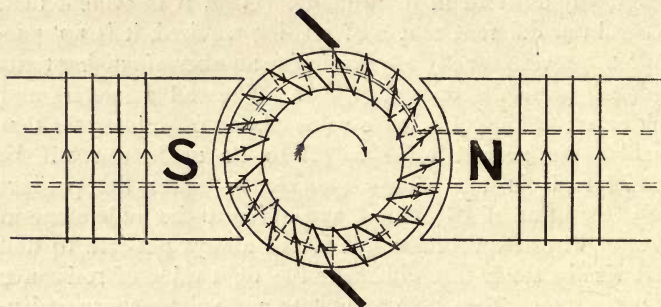


FIG. 15.

dynamo, with the wire on the field magnets and armature indicated by lines.

The direction in which the current flows in the field-magnet winding is indicated by arrows. By applying Dr. Fleming's rule for the direction of the E.M.F. in any wire on the armature, we are able to mark these wires to show the direction in which the current would flow if the brushes were connected. Thus, on connecting the brushes, a number of things will happen—

(1) Due to the resistance of the armature winding, we shall get a P.D. at the brushes which will be less than the whole generated E.M.F. by an amount which may be considerable at full load, though hardly noticeable at light load.

Thus, calling the whole E.M.F. = E , the current in the armature C_a , and its resistance r_a , we have that the value of the P.D. at the brushes—

$$e = E - C_a r_a$$

(2) In addition to this loss of volts in the armature resistance, we have to allow for the effect of the current in the armature winding as a M.M.F. acting in the magnetic circuit. Thus, with the brushes on the diameter of commutation, corresponding with no load, as shown in Fig. 15, we see that the armature winding and current produce a M.M.F., causing a magnetic flux at right angles to the flux set up by the magnets, and acting in a direction up the page, whilst the flux due to the magnet winding acts from right to left, the resultant flux through the armature will, of course, be a flux having a direction somewhere in between these two directions, or inclined to the horizontal line in the figure by an amount which will depend upon the relative magnitudes of the two M.M.F.'s.

Thus, if the armature current and number of windings be very large, we shall have the resultant magnetic flux taking a direction chiefly down the length of the page; the dynamo would then be a very bad one, and spark considerably. On the other hand, if the M.M.F. due to the magnet-field winding greatly predominates, we should have the resultant flux nearly horizontal as before taking any current from the armature. This state of affairs is that to be desired in all good dynamos and motors where sparkless collection among other advantages is wanted. In the best of machines, however, there is some distortion, and it is this we have next to consider. When a current is flowing in the armature the direction of the resultant flux will be as shown in Fig. 16, where a dotted line carried round the magnetic circuit indicates the path of the lines of force. The brushes have been adjusted so as to bear upon

a diameter at right angles to the direction of flux through the core; that is upon the new diameter of commutation. Two things will now be noticed—

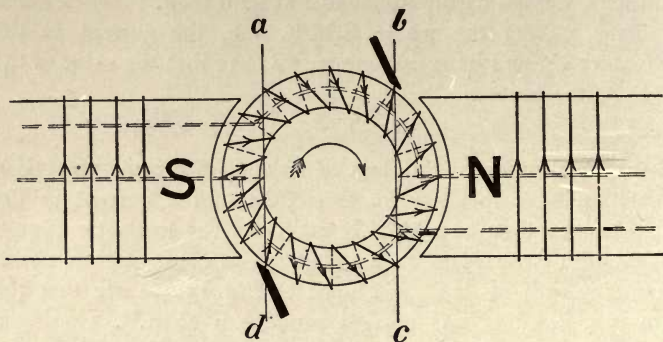


FIG. 16.

(1) The cross magnetic effect of the armature will now be somewhat decreased, being now only that due to the wires on the armature contained by the line *ad* or *bc*; and—

(2) The remainder of the winding on the armature, viz. that contained by the line *ab* or *dc*, is now a M.M.F. causing a magnetic flux in a direction parallel to the flux caused by the magnet winding, but in an opposite sense, or tending to directly decrease its magnitude. Since the value of the whole generated E.M.F. depends upon the value of the magnetic flux through the armature, this E.M.F. will no longer be as large as before a current was taken from the machine.

We must thus allow that the generated E.M.F. is now less than *E* by an amount, say *E*₁, and can write the terminal P.D.—

$$e = E - C_a r_a - E_1$$

In a great many cases we are required to produce a machine which will give a constant value of terminal P.D. *e*₁ at all loads, and in order to do this it will be necessary to make allowance for the reductions indicated in the above equation. In some cases, however, it is desirable to make a correction also for

the loss along leads, at the distant end of which lamps are to be kept at a constant P.D., and we may then add to the above losses the loss along the leads, or Cr_l , making now—

$$e = E - C_a r_a - Cr_l - E_1$$

Correction for these losses is commonly made by adding to the M.M.F. of the magnet-winding by adding a series coil to the shunt, thus compounding the machine. Now, such series winding has the effect, if connected as “long shunt,” of increasing the effective resistance of the armature, and our equation will now stand—

$$e = E - C_a r_a - C_s r_m - Cr_l - E_1$$

where r_m is the series-coil resistance. When, however, the connections are made “short shunt,” the series coil acts as an increase in the resistance of the leads, and the resultant reduction of terminal P.D. is slightly different, viz.—

$$e = E - C_a r_a - Cr_m - Cr_l - E_1$$

There is, however, a more important difference between the two cases: whereas the P.D. at the terminals of the shunt coil is practically constant in the case of “long shunt,” it is gradually increased from no load to full load with the “short shunt” connections, by an amount equal to the loss in the series coil, and thus the amount of series winding necessary in the case of “short shunt” is not quite so great as in “long shunt,” when other things are equal.

We have, then, that the generated E.M.F.—

$$E = e + C_a r_a + C_m r_m + Cr_l + E_1$$

putting C_m for the current in the series coil, which may be either C or C_a . Of these items we can have generally the most accurate knowledge, save in the case of E_1 . The exact effect of the band of winding contained by the lines ab or dc in Fig. 16, called the “demagnetizing band,” is somewhat difficult to predetermine. Its magnitude depends upon the position of the brushes, and this depends upon the exact distortion produced by the cross magnetizing band. This distortion can be found by plotting out on the drawing of the

machine the path of the lines for every value of the current in the armature, and thus finding the diameter of commutation for every value of the current. Considerable experience and care are required for this, and it will be out of place to further consider it here.

An example of the ascertainment of the value of the necessary series winding, when all the above losses have been found, will serve to sum up the matter.

A dynamo is required to give a terminal P.D. of e volts at all values of current, from zero to C ampères. In the first place, at very small output the whole generated E.M.F. need not be appreciably larger than e , and consequently a calculation must be made of the M.M.F. necessary to produce such a value of N that the generated E.M.F. = e , or—

$$N = \frac{e 10^8}{nw}$$

which will require a M.M.F. of \mathfrak{M} ampère-turns. The winding to produce this must be the shunt coil. In the second place, we have to allow that the losses at full load make the necessary generated E.M.F. to be E volts, and thus necessitate a greater value of magnetic flux, or—

$$N_1 = \frac{E 10^8}{nw}$$

to produce which the M.M.F. will be larger, and may be called \mathfrak{M}_1 .

Thus we have the additional M.M.F. required at full load is—

$$\mathfrak{M}_1 - \mathfrak{M} \text{ ampère-turns}$$

which must be supplied by the series coil in addition to that required to balance the demagnetization. The series M.M.F. is a product of the series turns into a current, which may be that in the main circuit (short shunt), or may be that in the armature (long shunt). In the latter event it will be noticed that when compounding has been accurately carried out for all the usual items, the P.D. at the terminals of the shunt coil is a constant at all loads (except when compounding for loss in

the leads). But in the short shunt combination the P.D. at the shunt terminals will only be e volts at very small loads, and will increase to $e + Cr_m$ volts at full load, thus increasing the M.M.F. and making consequently less series winding necessary. This alteration of M.M.F., due to the shunt coil, is of very great importance in all cases where there is imperfect compounding as any defect. Both over- or under-compounding is much exaggerated.

A little consideration will show that it is hardly possible to produce a perfectly constant P.D. at the terminals by compound winding. For if we calculate the additional ampère-turns to be supplied by the series coil in, say, two portions, viz. one for half-load, and the other as a further addition for the other half-load to full load, we shall find that the latter exceeds the former. Thus, let the extra M.M.F. required for half-load be \mathcal{M} ; this is due to an increase in the total flux necessary to increase the generated E.M.F. by an amount equal to the sum of a number of items, viz.—

$$C_a r_a + C_m r_m + C r_i$$

which will be all nearly enough half the amounts for full load, whilst the other item, E_1 , will be less than half, since it will be due to a demagnetizing band of less turns than at full load, and carrying only half the current. The items for the further increase up to full load will all be higher than for the first half-load, since the resistances will all be slightly higher, and also the demagnetizing band will have increased in number of turns as well as having been doubled in current; or \mathcal{M}_2 , the increase of magneto-motive force for the second half of full load, will be considerably greater than \mathcal{M}_1 . But both are produced by the increasing current flowing in coils of fixed number of turns, which will cause the additional ampère-turns to be simply proportional to the current. And also the relationship between the magnetic flux produced and the ampère-turns producing it is such as to make the additional effect of \mathcal{M}_2 actually less than \mathcal{M}_1 instead of greater.

It thus follows that if a dynamo is to give, say, e volts P.D. at its terminals at a certain speed at no load and also at full

load, it must give a P.D. which is greater than e at the intermediate loads.

Consequently it is customary to fix upon two points, say quarter-load and three-quarter-load, and so wind as to produce the same P.D. at the terminals at these points, letting the P.D. be slightly less than this at no load and full load, but slightly higher at half-load.

In the above considerations it is sometimes necessary to ask ourselves what effect in a composite magnetic circuit a given increase or decrease of M.M.F. will have. It is very easy to find what increase or decrease of M.M.F. a given increase or decrease of magnetic flux will require; but the converse is a much more difficult problem, since the relationship between all the items of the circuit is at once altered as soon as the flux is altered.

ARMATURE REACTIONS IN A MOTOR.

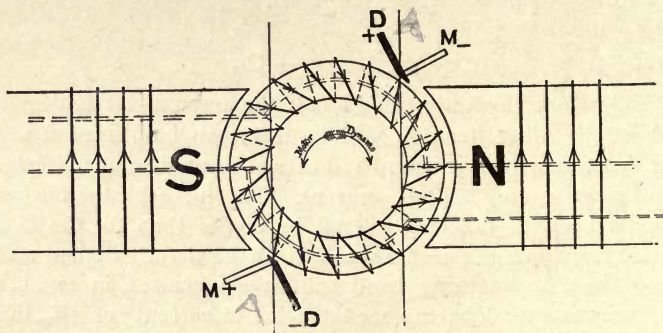


FIG. 17.

If in the above diagram DD represent brushes bearing on the commutator of a machine used as a dynamo, we shall have the current flowing as shown by the arrow when the direction of rotation is clock-wise, as indicated by the slope of the brushes. In order to cause rotation in the clock-wise direction, a pull will have to be supplied to the belt to overcome the drag between the current in the wires under the pole-pieces and the lines of force passing into and out of the armature. When,

however, the diagram is taken to represent a motor supplied with current to flow through the circuit in the direction as shown in the figure, this drag will now be the moving force, and the armature will revolve in a counter clock-wise direction, to admit of which the brushes MM will have to be used sloping in the opposite sense to the dynamo brushes, but bearing upon the same point on the commutator, since the nature of the magnetic forces acting in the magnetic circuit is still unchanged. But as the armature is now revolving in the opposite sense to what it was before the case, the E.M.F. generated will also be opposite to that previously generated, and will, consequently, be against the E.M.F. of the source supplying the motor with power, which will thus have to exceed that generated by the amount required for the resistance of the armature coils.

It thus follows that, as in a dynamo so in a motor, the current in the armature produces a demagnetizing effect upon the magnetic circuit. But whereas this demagnetizing effect is disadvantageous in a dynamo, it actually tends to aid constancy of speed in a motor by reducing the generated E.M.F. for a given speed somewhat in the proportion required by a constant E.M.F. of supply.

The above diagram is due to Mr. Swinburne.

CHAPTER IX.

EFFICIENCY.

THE ratio of the useful expenditure of energy in a system to the whole energy is called the "efficiency;" but we have, in the many and various cases to be considered, to sometimes separate the parts of a system and consider them independently, as will be seen in the examples taken.

As a simple case, we may take a single cell used to light a single lamp. Let the total E.M.F. of the cell be E volts, its internal resistance be r ohms, whilst that of the lamp is R ohms. Then the current flowing through the circuit will be—

$$C = \frac{E}{R + r}$$

and the rate of doing work in the lamp is—

$$C^2 R = \frac{E^2}{(R + r)^2} R$$

whilst the whole activity is—

$$C^2 (R + r) = \frac{E^2}{(R + r)^2} \times (R + r)$$

the efficiency is then the ratio of that part of the energy concerned in lighting the lamp to the whole activity, or is—

$$\frac{\frac{E^2}{(R + r)^2} R}{\frac{E^2}{(R + r)^2} (R + r)} = \frac{R}{R + r}$$

Or we may split our circuit up into items, and consider the ratio of the useful energy in the lamps to that in the lamps

plus that in the leads, thus getting the efficiency of distribution. Or we may consider the ratio of the energy given out by the cell to the whole energy produced by it, in which case we have the electrical efficiency of the cell or other machine.

Or, again, we may include the energy required to run the generator—a dynamo, for example—against the various sources of friction, and thus state the efficiency as the ratio of the energy delivered from the terminals to the energy put in at the pulley, thus getting the mechanical efficiency, or commercial efficiency.

In the case of motors, we can take the ratio of the power at the pulley to the power supplied at the terminals, which is again the commercial efficiency. On the other hand, we may regard the motor as a machine for the conversion of electrical energy into mechanical energy, whether useful or not, and so obtain the electrical efficiency.

With secondary cells we may, on the one hand, regard the cell as a store of quantity of electricity, and state the quantity efficiency as the ratio of the coulombs got out to the coulombs put in; or we may consider the cell as a store of energy, and state the energy efficiency as the ratio of the watt-hours got out to the watt-hours put in.

Also, as a dynamo and engine will be required for charging, we may have a combined efficiency of the whole apparatus, called the plant efficiency, and being the ratio of the watts got out to the power required to run the dynamo stated in watts.

I. Efficiency of Distribution and Transmission.

(a) In a simple parallel system, with all the lamps at the ends of the leads, if r_l = resistance of the leads, C the current through the lamps, and e the P.D. at the lamps, we have—

$$\begin{aligned} \text{energy used in the lamps} &= eC \\ \text{,, wasted in leads} &= C^2 r_l \end{aligned}$$

and the efficiency will be—

$$\frac{eC}{eC + C^2 r_l} = \frac{e}{e + Cr_l}$$

which is the same thing as the ratio of the useful P.D. to the total P.D.

(b) Simple parallel system, with the lamps uniformly distributed along the length of the leads. Here the loss of volts in the leads will be—

$$\frac{C}{2}r_l$$

making the energy waste in the leads = $\frac{C^2}{2}r_l$.

But the energy expended in the lamps will be—

$$C \times \left(e + \frac{C}{2}r_l \right) \text{ as an average}$$

where e is the P.D. at the far end; or may be put more conveniently in terms of the initial P.D., thus—

$$C \times \left(E - \frac{C}{2}r_l \right) \text{ average}$$

where the efficiency is—

$$\frac{C \left(E - \frac{C}{2}r_l \right)}{CE}$$

(c) A parallel system with feeders. Let r_f be the resistance of the feeders, and r_d be the resistance of the distributors between two feeding centres. Then the total current will be, say, C ampères in the feeders, or an average of $\frac{C}{4}$ ampères in the distributors.

The loss of energy in the feeders will then be C^2r_f , whilst that in the distributors will be—

$$\frac{C^2}{16}r_d$$

If also E be the P.D. at the starting ends of the feeders, the mean P.D. at the lamps will be—

$$E - Cr_f - \frac{C}{4}r_d$$

whence the efficiency will be—

$$\frac{C \left(E - Cr_f - \frac{C}{4}r_d \right)}{CE}$$

Or, again, the ratio of average P.D. at lamps to P.D. at the dynamo.

(d) Series circuit containing n items, each of r ohms resistance, and having a constant current of C ampères. Calling r_l the resistance of the leads, the efficiency is—

$$\frac{C^2 nr}{C^2(nr + r_l)}$$

again the ratio of the P.D. at the lamps to the total P.D.

II. Efficiency of dynamos, motors, etc.

(a) Magneto-dynamo. Here the only loss is in the armature and brush contacts, etc., the circuit being a simple series system. Let e be the total P.D., and r_a the resistance of the armature, etc.; then the output is eC , and the waste in the armature $C^2 r_a$, whence the—

$$\text{electrical efficiency} = \frac{eC}{eC + C^2 r_a}$$

which is also the ratio of the useful P.D. to the whole P.D.

From a theoretical point of view the efficiency of a magneto may be higher than that of a dynamo since there is no waste of energy to magnetize the field-magnets.

The other losses in the machine will consist of M the mechanical friction, H the hysteresis loss, and F the eddy currents. Calling all these W_1 , we have the—

$$\text{mechanical efficiency} = \frac{eC}{eC + C^2 r_a + W_1}$$

(b) Series dynamo. Calling as before e , C , and r_a respectively the terminal P.D. current and armature resistance, and r_m the resistance of the magnet coils, we have the—

$$\text{electrical efficiency} = \frac{eC}{eC + C^2(r_a + r_m)}$$

And again, if W_1 stands for all the other losses expressed in watts, then the—

$$\text{mechanical efficiency} = \frac{eC}{eC + C^2(r_a + r_m) + W_1}$$

(c) Shunt dynamo. Using the same symbols as before,

and calling the resistance of the shunt coil r_s , we have the output is eC , but the current in the armature is greater than C by the amount taken by the shunt coil, viz. $\frac{e}{r_s}$, the waste in the armature being consequently—

$$\left(C + \frac{e}{r_s}\right)^2 r_a$$

Whilst the shunt-coil waste is $\frac{e^2}{r_s}$, and the—

$$\text{electrical efficiency} = \frac{eC}{eC + \frac{e^2}{r_s} + \left(C + \frac{e}{r_s}\right)^2 r_a}$$

And W_1 having the same meaning as previously, the—

$$\text{mechanical efficiency} = \frac{eC}{eC + \frac{e^2}{r_s} + \left(C + \frac{e}{r_s}\right)^2 r_a + W_1}$$

(d) Compound dynamo, short shunt. Here the output is eC , as before. The current in r_m is C amperes, but that in r_a is $C + \text{shunt current}$, or—

$$C + \frac{e + Cr_m}{r_s}$$

whence the various losses are $C^2 r_m$ in the series coil,

$$\frac{(e + Cr_m)^2}{r_s}$$

in the shunt coil, and—

$$\left(C + \frac{e + Cr_m}{r_s}\right)^2 r_a$$

in the armature, making the electrical efficiency—

$$= \frac{eC}{eC + C^2 r_m + \frac{(e + Cr_m)^2}{r_s} + \left(C + \frac{e + Cr_m}{r_s}\right)^2 r_a}$$

the mechanical efficiency being—

$$= \frac{eC}{eC + C^2 r_m + \frac{(e + Cr_m)^2}{r_s} + \left(C + \frac{e + Cr_m}{r_s}\right)^2 r_a + W_1}$$

(e) Compound dynamo, long shunt. The current in the shunt is $\frac{e}{r_s}$, and flows through both r_m and r_a , in addition to the main-circuit current C ;

whence the—

$$\text{electrical efficiency} = \frac{eC}{eC + \frac{e^2}{r_s} + \left(C + \frac{e}{r_s}\right)^2 (r_m + r_a)}$$

and the—

$$\text{mechanical efficiency} = \frac{eC}{eC + \frac{e^2}{r_s} + \left(C + \frac{e}{r_s}\right)^2 (r_m + r_a) + W_1}$$

(f) A series motor. The electrical efficiency is the ratio of that part of the energy supplied to the machine, which is actually converted into mechanical work, whether useful or waste, to the whole energy supplied. Calling E the P.D. of supply, with C the current, and r_a and r_m having the values above, we have that the back E.M.F. of rotation is—

$$\epsilon = E - C(r_a + r_m)$$

and the whole rate of doing work mechanically is $C\epsilon$;
whence the—

$$\text{electrical efficiency} = \frac{C\epsilon}{EC} = \frac{\epsilon}{E}$$

The mechanical efficiency is—

$$\frac{EC - C^2(r_a + r_m) - W_1}{EC}$$

or—

$$\frac{C\epsilon - W_1}{EC}$$

Since of the total rate of doing work mechanically the part W_1 being internal friction, etc., is wasted; M is direct mechanical friction, H is molecular friction, but F is a load on the armature of exactly the same nature as a closed secondary on a dynamotor armature.

(g) Shunt motor. Calling E the P.D. at which a current of C ampères is supplied, we have that of C ampères, the shunt coil takes $\frac{E}{r_s}$, leaving the—

$$\text{current in the armature} = C - \frac{E}{r_s}$$

$$\text{due to which the loss in } r_a \text{ is } \left(C - \frac{E}{r_s} \right) r_a \text{ volts}$$

$$\text{leaving the B.E.M.F.} = \epsilon = E - \left(C - \frac{E}{r_s} \right) r_a$$

whence the—

$$\text{electrical efficiency} = \frac{\left\{ E - \left(C - \frac{E}{r_s} \right) r_a \right\} \left(C - \frac{E}{r_s} \right)}{EC}$$

and the mechanical efficiency

$$= \frac{\left\{ E - \left(C - \frac{E}{r_s} \right) r_a \right\} \left(C - \frac{E}{r_s} \right) - W_1}{EC}$$

(h) Compound motor, short shunt. Calling EC the supply as before, we have the shunt current is—

$$\frac{E - Cr_m}{r_s}$$

Whence the armature current is—

$$C - \frac{E - Cr_m}{r_s}$$

and the B.E.M.F.—

$$\epsilon = E - Cr_m - \left(C - \frac{E - Cr_m}{r_s} \right) r_a$$

and therefore the electrical efficiency is—

$$\frac{\left\{ E - Cr_m - r_a \left(C - \frac{E - Cr_m}{r_s} \right) \right\} \left(C - \frac{E - Cr_m}{r_s} \right)}{EC}$$

whilst the mechanical efficiency—

$$= \frac{\left\{ E - Cr_m - r_a \left(C - \frac{E - Cr_m}{r_s} \right) \right\} \left(C - \frac{E - Cr_m}{r_s} \right) - W_1}{EC}$$

(k) Compound motor, long shunt.

Here the shunt current is $\frac{E}{r_s}$

and the B.E.M.F.—

$$\epsilon = E - \left(C - \frac{E}{r_s} \right) r_m - \left(C - \frac{E}{r_s} \right) r_a$$

and the electrical efficiency

$$= \frac{\left[E - \left\{ \left(C - \frac{E}{r_s} \right) (r_m + r_a) \right\} \right] \left(C - \frac{E}{r_s} \right)}{EC}$$

whilst the mechanical efficiency

$$= \frac{\left[E \left\{ \left(C - \frac{E}{r_s} \right) (r_m + r_a) \right\} \right] \left(C - \frac{E}{r_s} \right) - W_1}{EC}$$

(l) Secondary battery. Calling the charging current C ampères and the time of charging t hours, the corresponding quantities for discharging being c_1 and t_1 , we have the ampère hours put in $= ct$ and output $= c_1 t_1$, whence the—

$$\text{quantity efficiency} = \frac{c_1 t_1}{ct}$$

and may be as much as 100 per cent.

Considering, however, the fact that the B.E.M.F. of the cell is higher on charging than the available E.M.F. for discharging, we have the energy put in is—

$$Cte$$

and that got out is $C_1 t_1 e_1$ when e and e_1 are respectively the E.M.F. to charge against a certain B.E.M.F. and resistance, and the P.D. available for use at the terminals on discharging. The

$$\text{energy efficiency is} = \frac{C_1 e_1 t_1}{Cet}$$

(m) Plant efficiency. This must include all sources of loss and give the ratio of—

The useful work done in a given time

The energy supplied to the prime mover for the same time

CHAPTER X.

VOLTMETERS AND THEIR RESISTANCE COILS.

IN the following examples notice will be taken of the effect of temperature on the readings of voltmeters of the magnetic class. As was mentioned in the chapter dealing with the winding of coils, there is no difference between an ammeter and a voltmeter in point of shape, etc. There is, however, this important difference between them—that, whilst the former has its dial graduated in terms of the current passing through its coil, the dial of the latter is marked in terms of the product of the current through the coil into the resistance of that coil, such resistance being taken at the temperature at which the instrument was calibrated. Thus the readings of an ammeter are independent of the value of its resistance, but those of a voltmeter depend upon the resistance of its circuit, and will only be correct when that resistance has the value it had when the dial was marked. It is, therefore, the object of the instrument-maker to so contrive matters that the resistance of the voltmeter circuit shall vary as little as possible. Alteration in temperature may be produced in two ways, viz. by external means such as atmospheric variation, or variation due to locality; or by internal means such as the expenditure of energy in the coil due to the current through it. We will call the first of these the “temperature” error, and the second the “heating” error. The employment of a material such as manganin or constantan, will make both these errors quite small, or even negligible as far as mere temperature variation of resistance (T.V.R.) is concerned, though the actual heating may be very large, and even dangerous. For the energy waste

in a coil of a given size producing a given value of ampère-turns will be directly proportional to the resistivity of the material of the wire. When the resistivity is very high, it is not easy to make an instrument coil of reasonable dimensions without excessive heating and waste of energy, and owing to the want of a material of low specific resistance and small or negative T.V.R., we are obliged to make a compromise between the two extremes, and wind that part of the instrument which is to produce the magnetic effect with wire of small resistivity in order to get large ampère-turns with small heating; and the rest with wire of small T.V.R., and having with advantage a high resistivity. The two things to be aimed at are, firstly, to get the requisite M.M.F. with small energy waste, and then to get the necessary freedom from variation of resistance by addition to the instrument resistance of a coil having a constant, or fairly constant, value of resistance. This is effected by making the working coil of the instrument of copper wire to get the maximum M.M.F. with the least energy waste, and by adding to the instrument so wound an extra coil or resistance of, say platinoid or manganin, materials having very small coefficients of T.V.R., and thus causing the effective total resistance to remain nearer a constant than if copper alone had been employed. The effect of so adding to the resistance of the working coil is twofold; firstly, the current through the instrument due to a certain P.D. will be decreased, and consequently the deflection for a given number of volts will be less than before, or the total reading of the instrument will be increased; and, secondly, there is the tendency to greater constancy in total resistance. The increase of total reading power is obviously proportional to the increase of resistance; thus if the extra coil have a resistance equal to that of the instrument's working coil, the instrument will read twice as much; or, more generally, if the extra coil be n times the resistance of the working coil, the total reading power will be $(n + 1)$ times the original. The T.V.R. of the whole combination is, on the other hand, inversely as the number of times the total resistance is increased if the material of the extra coil has no T.V.R. If the T.V.R. of the extra coil be

important, having a value c say, whilst the T.V.R. of the working coil is b , and the extra coil has a resistance n times that of the working coil then the combined T.V.R. will be—

$$a = \frac{b + nc}{n + 1} \text{ per cent. per } 1^\circ \text{ C.}$$

when b and c are stated in values per cent. per 1° C. The plus sign is mostly the case, but when carbon or manganin is employed for the extra resistance, then c is negative, and by adjusting n so that—

$$nc = b$$

we shall obtain a system of perfectly constant resistance.

It thus follows that the value of an indication on a voltmeter in volts is found by multiplying the reading as marked on the dial by the ratio of the present resistance total to the total resistance when calibrated, or—

$$\text{volts} = \text{deflection} \times \frac{\text{present resistance}}{\text{resistance when calibrated}}$$

The deflection being of course marked by a number which is equal to the product of the resistance of the instrument when calibrated into the current which a given P.D. produces through the instrument.

The value of the present resistance depends upon the temperature and the T.V.R., values for which are given on A, p. 209.

Thus let R_τ be the resistance at which the instrument was calibrated (temperature, say $\tau^\circ \text{ C.}$), and let R_t be the resistance at some other temperature $t^\circ \text{ C.}$, which may be either higher or lower than $\tau^\circ \text{ C.}$ Let a be the T.V.R. per cent. per 1° C. Then, nearly enough for practical purposes—

$$R_t = R_\tau \left\{ 1 + \frac{a}{100}(t - \tau) \right\}$$

when the difference $t - \tau$ may be zero, positive when t is greater than τ , or negative when t is less than τ .

Similarly, the value of any indication on the voltmeter will be always proportional to its resistance at the time of taking

a reading, and if we call the value per one division k volts, when the instrument is at the correct temperature we shall then have that the value of one division at any other temperature will be—

$$K = k \left\{ 1 + \frac{\alpha}{100}(t - \tau) \right\}$$

CHAPTER XI.

ELECTRIC TRACTION, RAILWAYS, ETC.

THE application of the electric motor to tramways and other tractive work renders it necessary to ascertain the horse-power required for any particular case, and the data necessary for this purpose may now be discussed. In order to draw a car along a line, the mechanical pull required will depend upon the following things—

- (1) The weight of the car.
- (2) The condition of the line.
- (3) The inclination of the line.

By the condition of the line is meant its smoothness and freedom from dirt or other obstruction. In the best possible cases of railway work the pull required to keep a ton weight of car in motion on a level line may be less than 10 lbs., but in some cases, even with grooved rails, the pull may be 15 to 20 lbs. in good cases, and even 40 lbs. or more in cases where dust or road grit has collected in the rails. Curves also cause a local increase of resistance to traction.

In order to allow for the inclination of the line we have to take into consideration the effect of gravity, which effect may be against us in going up an incline, but which is for us in going down an incline. It is seldom that the inclination is so much as to render it necessary to distinguish between the length of the actual distance gone and the true horizontal distance, and consequently we may write the pull due to gravity as being—

where n is the number of feet along the line for a rise or fall of 1 foot, this pull being + or - according as the inclination is up or down. We shall thus have the pull required to move a car of W tons weight along a line of 1 in n inclination with a resistance to traction of x lbs. per ton as being in pounds—

$$P = Wx \pm \frac{W2240}{n}$$

taking the + sign when going up hill and the - when coming down hill.

Again, the rate of doing work is proportional to the speed of running, and we have the horse-power required will be the pull multiplied by the distance gone through in feet per minute and divided by 33,000. Thus, if S = the speed in miles per hour, we have the horse-power is—

$$\text{H.P.} = \frac{88PS}{33000}$$

and this is the useful mechanical rate of doing work, and to it must be added that required to run and excite the motor and that required to transmit the power of the motor to the axle of the car.

In general the conditions are so disadvantageous to the high-speed motor, that in practice it is found that for every horse-power put into the motor-terminals we may get out from 60 to 80 per cent. as useful work to run the car; the lower figure is frequently exceeded, but the higher can only be attained under the best conditions, and with large motors. Calling this factor $\frac{100}{y}$, we have that the electrical power put into the motors must be—

$$\frac{100}{y} \left(\frac{88PS}{33000} \right) \text{ or, in full—}$$

$$\text{H.P. required} = \frac{100}{y} \left\{ \frac{88S \left(Wx \pm \frac{W \times 2240}{n} \right)}{33000} \right\}$$

which is converted into watts by multiplying by 746. Now, this power is a product of volts and ampères, and it is simply

a matter of convenience as to what the E.M.F. may be. The motor is in any case arranged with its gearing so that a given current through its armature means a given pull for traction, and the value of this pull per ampère can usually be stated or fixed. From this and the actual pull required we can readily find the current required to move the car, and can add that to overcome internal friction, etc. From the current and the speed of running we can next find the useful rate of doing work, and thence the B.E.M.F. of the motor. In the case of shunt motors this B.E.M.F. will be practically constant, being in any case proportional to the speed, and the cars will run at a nearly constant maximum speed both up and down hill, and on the level if supplied with a constant E.M.F. A knowledge of the electrical data will then enable us to ascertain the necessary increase of volts on account of internal losses, and to state the exact value of the P.D. of the mains, In some cases the tractive pull per ampère is not a constant, since with series motors the drag on the wires is increased by an increase of current both on account of that increase itself and also on account of the consequent increase of magnetic flux through the armature. When series motors are run off mains at a constant P.D., we shall have their speeds varying by an amount which is greater than in the case of shunt motors, having a practically constant excitation. Thus, at places on the line taking a heavy load, the extra current in the magnet coils will cause the speed to go down, since otherwise the B.E.M.F. of the motor would soon exceed that of the supply mains, were that possible. The tractive force being thus increased in a proportion more nearly as the square of the current, makes the series motor particularly suitable for overcoming steep gradients and other heavy loads with less heating and waste of energy than would be the case with a shunt motor, for the series motor will automatically reduce its speed up hill, and by increasing its field strength take less increase of current to produce the necessary tractive pull.

When getting up speed the car has to store an amount of energy, equivalent in foot pounds to half its mass, multiplied by the square of its velocity in feet per second. The extra

pull required to do this will depend upon the time taken to attain the speed, and we can write the work to be stored in foot pounds, as—

$$\frac{W \times 2240}{2g} \left(\frac{S \times 5280}{3600} \right)^2 = K$$

When g is taken as 32, we can simplify this to—

$$K = WS^2 \times 75.3 \text{ foot pounds}$$

where W = tons weight of car and S the speed to be got up in miles per hour.

Now, if this store of energy is to be acquired in m , minutes, we must remember that the car in gradually getting up the speed of S miles per hour will go through a distance of—

$$m \times \frac{S \times 5280}{60 \times 2} = l \text{ feet say}$$

since its average speed during the time will be $\frac{S}{2}$ if the acceleration be uniform. And through this distance a pull over and above that required to overcome the tractive resistance will be required of—

$$\frac{K}{l} \text{ pounds}$$

or the extra pull during the time of m , minutes, to get up speed of S miles per hour will be—

$$\frac{WS^2 \times 75.3}{44Sm} = \frac{WS}{m} \times 1.711 \text{ pounds}$$

Or if speed is to be attained in t seconds, the extra pull is—

$$\frac{102.7 WS}{t} \text{ pounds}$$

If also p be the pull per ampère, the current required will then be—

$$\frac{C = Wx + \frac{W2240}{n}}{p} \text{ ampères}$$

to which must be added the extra current to get up speed, or—

$$C_1 = \frac{102.7 WS}{t\phi} \text{ ampères}$$

The value of ϕ the pull per ampère may be nearly a constant for all values of current in the case of a shunt motor; but in the case of a series motor it has more nearly the value—

$$\phi = \frac{\frac{n}{a} C^2}{b + C}$$

where n = speed per minute and a and b are constants, depending upon the size of the machine.

In questions of electric traction involving a knowledge of the value of the pull produced by a given current in the armature, or of finding the value of the current required to give a certain pull, we must be careful to distinguish between the shunt, compound, and series types of motors.

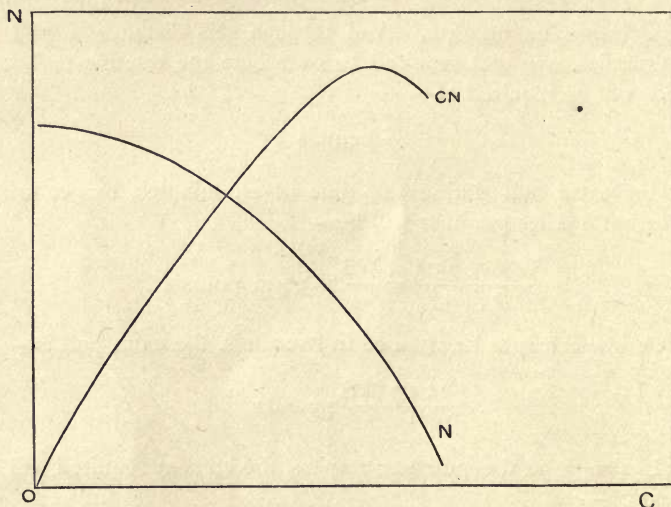


FIG. 18.

Shunt Motor.—When the E.M.F. of supply is constant, we may regard the magnetizing force, acting upon the magnetic

circuit as practically constant, subject to the fact that distance from the generating station will mean a lowering of the P.D. at the terminals of the shunt coil by the ohmic loss in the leads. The pull or torque is always proportional to the product of current and lines total; and these last should be constant in a shunt motor with constant P.D. at the terminals were it not for the fact that armature reactions produce a demagnetizing effect, and the curve of relationship between N and C , the current through the machine will be of the nature of the curve shown in Fig. 18, and consequently the pull per ampère will be decreasing slightly as the current increases. Thus at times of very large current, as at starting, the actual pull may not be very large, and the maximum pull or torque may never be great.

Compound Motor.—Taking next the case of a motor compounded for ordinary work, that is for constant speed with varying load, we find that the effect of the compounding series coil is to still further increase the demagnetizing effect of the current, with the result that the curve of relationship between N and C will be still more drooping at high values of C than that shown in Fig. 18, with the effect of a still further reduced pull under similar conditions.

But when the series-magnet winding is connected so as to magnetize the circuit, we may have this effect made, either to give a constant value of N for all values of C (in which case there will always be a constant pull per ampère, whatever value the current may have), or to actually produce an increase of lines with an increase of current, making the curve of $N - C$ rise for an increase of C , and thus giving a pull per ampère increasing with the current. It will not be necessary to investigate this action in combination with the shunt coil, but to go on to consider the effect of a series coil alone.

Series Motor.—Here the value of N is constantly increasing with the current, and would have a curve of relationship similar to the βH curve for a composite magnetic circuit, were it not for the fact of demagnetization causing a tendency to drop at high values of C . Taking the shape of NC , as shown in Fig. 19, we may also plot a curve, showing the relationship



between C and the product CN, as shown in the upper curve. The pull exerted is directly proportional to the ordinate of the second curve, and can be found from the nature of the winding

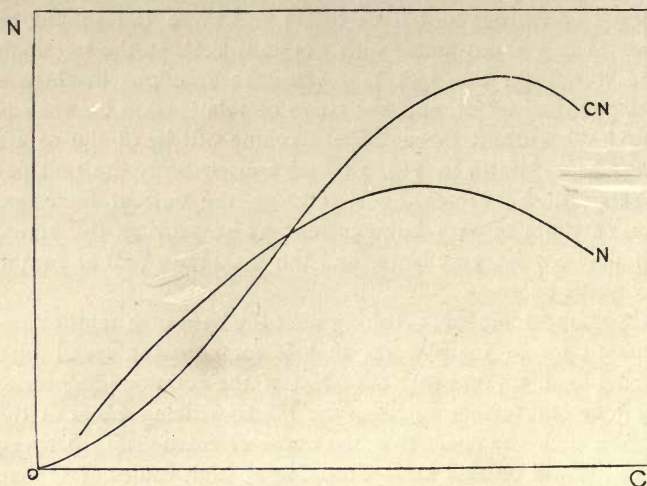


FIG. 19.

and gearing employed. Calling the total pull for tractive effect, $p = aCN \times 10^{-6}$, where a is a constant depending upon the type of machine, we have from a knowledge of the value of p in pounds that the product—

$$CN = \frac{p \times 10^6}{a}$$

and a reference to the curve will give the value of C and N corresponding with this product.

It is also clear that the pull will reach a maximum value, and the performance of the motor is thereby limited.

CHAPTER XII.

ALTERNATING CURRENT CIRCUIT.

WHEN the current in a circuit changes periodically in direction, passing through a set of cyclic changes of value, it is termed an alternating current, and, owing to the disturbing influence of the medium which surrounds the circuit, the usual relationship known as Ohm's Law requires modification before it can completely represent the true state of affairs. The manner of change of value of the current from time to time is different in different types of machinery, but is very frequently of a nature simply expressed as a sine function of the time. Such a current can be represented in all possible values by the curve in Fig. 20, where ordinates above and below the line Ot represent the instantaneous values of the current in amperes corresponding with any instant in time marked along Ot . Such instantaneous values can be represented by the expression—

$$C_i = C_m \sin \frac{2\pi}{\tau} t$$

where C_i is the value at the instant in time t , C_m the maximum value to which the current ever attains, and τ is the time in seconds taken to go through one complete cycle, or change from zero rising to a positive maximum, declining to zero, reversing and rising to a negative maximum, again declining to zero. The curve can be drawn from a table of natural sines by dividing the horizontal line into 360 parts corresponding to the 360 degrees in a complete revolution, and marking points at a distance above or below the horizontal equal to C_m times the natural sine of the angle. The circular measure of 360

degrees is 2π , and it thus follows that when t has such values as to make $\frac{t}{\tau} \times 2\pi$ equivalent to convenient angles, the curve may be easily drawn in. When $t = \tau$ the angle $= 2\pi = 360^\circ$.

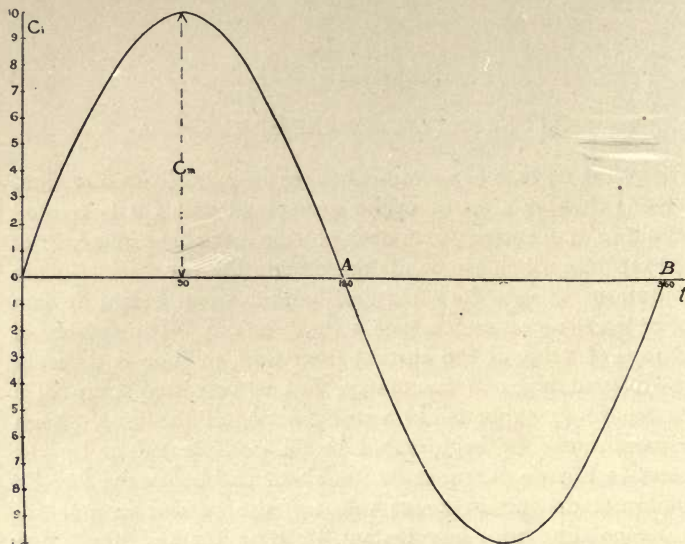


FIG. 20.

The time τ is called the time of one complete cycle or period, and ranges with different makers from $\frac{1}{20}$ second to $\frac{1}{130}$ second, though smaller values were used in past times. It has a very general value in English practice of either $\frac{1}{80}$ or $\frac{1}{100}$ second, the value $\frac{1}{130}$ being American. It is thus customary to describe the circuit as one having, for example, 100 periods per second; or to say it has a frequency or periodicity of so and so per second. Calling the frequency p periods per second, we may put instead of the above expression—

$$C_t = C_m \sin 2\pi p t$$

where p stands for $\frac{1}{\tau}$.

Now, it will be quite evident that in some cases the rapidity with which the current changes in value will materially affect the conditions of working; and it will be necessary to investigate as fully as possible the effect of any such disturbance for the different conditions possible. There are four ways in which an electric current can become manifest to us—

- (1) Chemical.
- (2) Thermal.
- (3) Magnetic.
- (4) Electrostatic.

And we will examine the effect produced for each of these considered separately.

I. The Chemical Effect.—With continuous or uni-directed currents the chemical change produced is always proportional to the strength of the current and the time during which it flows. It is also dependent upon the direction of flow, being of the nature of a solution where the current enters, and a deposition where the current leaves the circuit. Thus, if a current of one ampère be passed for the space of time of t seconds through a solution of copper sulphate, it will dissolve from the plate of copper at which it enters the solution, and deposit upon the plate at which it leaves the solution, an amount of copper which may be expressed as—

$$t \times 0.00032959 \text{ gramme}$$

Thus, during the time from 0 to A in Fig. 20, we have the current flowing all in one direction; and if such current be passed through a solution of copper sulphate, it will cause a deposit of copper on one plate and a solution of copper from the other plate, which will be in amount equal to the product of the average strength of the current into the time, viz.

$\frac{\tau}{2}$ seconds, and into the numeric used above (the electrochemical equivalent of copper).

But if the circuit be not broken at the instant when the current becomes zero, the contrary effect will be produced, and there will now be a re-solution of copper from the plate which before received the deposit, and a deposition on the plate from

which the copper was dissolved; and this reverse effect will be of exactly the same magnitude as the previous effect if the current be maintained for the next or second half-period.

It thus becomes apparent that an alternating current cannot produce any chemical effect; for what is done during any half-period is exactly undone during the next.

When, however, the current is "rectified" by a commutating device, timed so as to exactly reverse the connections at the instant the current changes in direction, we shall get a total chemical effect which will be proportional to the product of the time and the average value of the current, or is—

$C_e \times 0.00032959$ grammes of copper

Now, it can be shown, by carefully measuring the area of the half-wave from 0 to A, that its average height is 0.6366 times C_m the maximum height, and we thus have that the amount of copper deposited by an alternating current which has been rectified is—

$0.6366 \times C_m \times 0.00032959 \times t$ grammes

and knowledge of the electro-chemical equivalent of any other material will similarly give the sum-total of the effect produced, as, for instance, in charging a secondary battery.

II. Thermal Effect.—This is, at any instant, proportional to the rate of expenditure of energy, and is independent of the direction of the current; thus, it is proportional to—

$C_i^2 R$

In order to find an expression for the average effect so produced, it is necessary to draw a curve which represents the quantity $C_i^2 R$, and to find the average height of such a curve. Thus, if $C_m = 10$ ampères, and R be 1 ohm, this curve of energy expenditure will have zero values coinciding with the zero values of current, and will rise to a maximum of 100 at the time when the current is maximum. If now the length corresponding to half a wave be divided up into 18 parts, and the area of each such slice be taken, all such areas being added, it will be found to amount to 899.6, when the width of each slice is called unity, and its height taken as that of its middle

value as marked on the scale of the diagram. The average height is consequently—

$$\frac{899.6}{18} = 49.95$$

which would have come out = 50 exactly, if we had been able to estimate values more exactly. Now, this 50 is the average rate of expenditure of energy in the circuit, and consequently is of the form—

$$C^2 R$$

but we know R to be unity, whence it follows that the C^2 must be = 50, or the effect produced is the same as would be the case had a current of magnitude—

$$\sqrt{50} = 7.071 \text{ ampères}$$

been maintained constant for the corresponding time. But this current is—

$$0.7071 \times \text{the maximum}$$

and is also the—

$$\sqrt{(\text{mean square})}$$

It is called the “virtual” value, and is really the value about which we should know.

As far, then, as working in the circuit in heating it, the alternating current which follows a sine law has an effect which is the same as would be produced by a continuous current having a magnitude of—

$$0.7071 \times C_m$$

and lasting for the same length of time.

From this value the maximum can be found by multiplying the virtual by $\frac{1}{0.7071}$, or 1.414, or—

$$\text{max. } C = C \text{ virtual} \times 1.414$$

Before going on to consider the magnetic effect in detail, it will be as well to point out that an electro-dynamometer will be affected by the value of the square of the current at any instant as with continuous currents, and will thus read a quantity which is this $\sqrt{\text{mean square}}$, or virtual value.

It follows from this that the rate of expenditure of energy in heating a circuit will be proportional to the product of the resistance in ohms into the square of the electro-dynamometer reading.

And also, when the current has been rectified, the electro-dynamometer will still read the virtual value, or—

$$0.7071 \times \text{maximum}$$

But it has been shown that the chemical effect produced by a rectified current is only proportional to—

$$0.6366 \times \text{maximum}$$

whence it follows that the dynamometer will indicate a value which is—

$$\frac{0.7071}{0.6366} \text{ or } 1.11 \text{ times too much}$$

III. The Magnetic Effect.—This may take place under circumstances which may be summed up in the following order, beginning with the condition where no magnetic effect can be produced:—

(a) No magnetic effect possible; circuit non-inductive.

(b) Magnetic effect set up in a medium which has a uniformly constant permeability—say air where $\mu = 1$ —and which is also non-conducting.

(c) Medium having constant $\mu = 1$, but being conducting.

(d) Medium having variable value of μ , as in a closed iron circuit. Non-conducting, *i.e.* laminated.

(e) Medium with μ variable but conducting; closed circuit.

(f) Medium with μ variable, but of composite character, as in the case of an open-iron circuit. Non-conducting.

(g) Medium with variable μ , composite and conducting.

For the sake of simplicity, capacity is supposed to be absent in all the above cases.

(a) Non-inductive circuit. Here the only effect which is possible has already been considered in the section on the thermal effect. It will be as well, however, to complete that case by considering the value of the E.M.F. in the circuit.

In order to cause a current of C_i ampères to flow in a circuit having a resistance of R ohms, we know that an E.M.F. of—

$$E_i = C_i R \text{ volts}$$

will be necessary, and this being the only E.M.F. in the circuit, will necessarily be always proportional to the current, and is then said to be in phase with the current, having its zero values and the positive and negative signs at the times when the current has these values and signs.

When this is the case, we can represent the rate of expenditure of energy in the circuit by the product of the readings of an electro-dynamometer and an electro-static voltmeter, both of which instruments give the virtual or $\sqrt{\text{mean square}}$ values. Thus—

$$\text{watts} = C_v^2 R = C_v E_v$$

where C_v and E_v stand for the virtual values, that is are 0.7071 times the maximum.

(b) Magnetic effect when the medium surrounding the circuit is, say air, having $\mu = 1$ and constant, and also being non-conducting.

Consider three rings superposed, the middle one carrying an alternating current. These rings are represented in section in Fig. 21.

The middle ring has a current flowing in it of a kind represented in Fig. 20, and consequently rising in a certain direction which we will call positive, when it is as indicated by the arrow in Fig. 21, where the middle ring has then a polarity of N face at the top and S face at the under side. Imagine the current growing from 0 upwards; as it grows lines of force will increase in number, threading through the upper ring so as to enter its section inwards from all points round it. An effect will consequently be produced which will be of the same kind as if the upper ring were being dropped upon the middle ring when that is producing a constant magnetic flux. By Fleming's rule it is easy to see that there will be an E.M.F. set up in the upper ring, having a direction contrary to that in the middle ring, or as shown by the arrows; and this E.M.F. will be greatest where the rate of change of lines is greatest, and will

be least, or zero, when the rate of change of lines is zero. Thus, it will be greatest at the moment the current in the

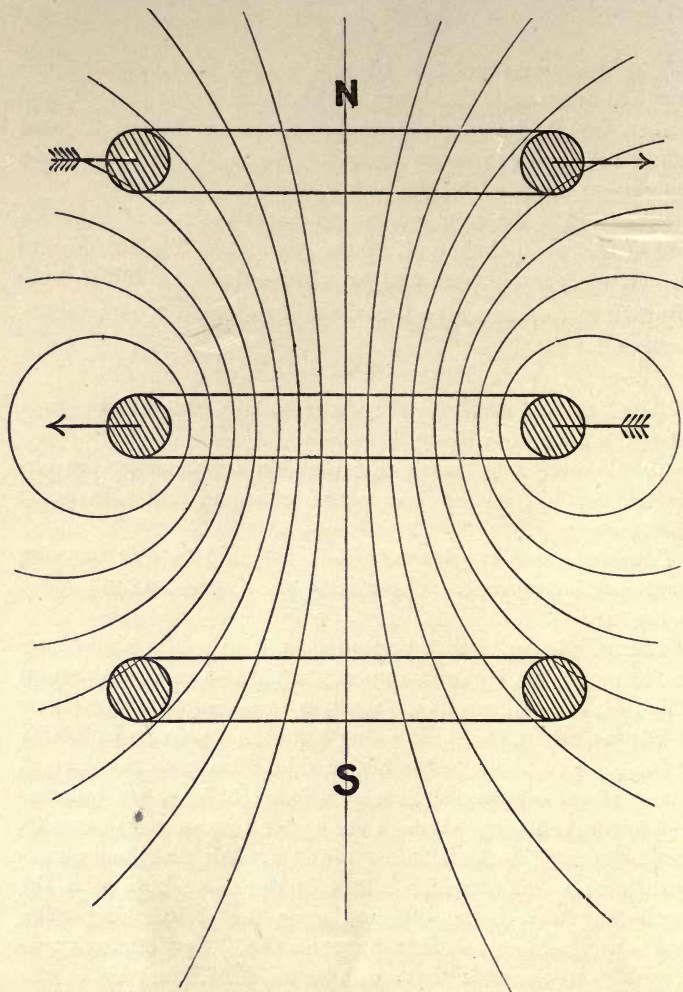


FIG. 21.

middle ring starts, and will be zero at the moment when that current has attained the maximum value.

Again, when the maximum has been passed and the current in the middle ring is now decreasing, there will be produced in the upper ring an effect of the same kind as if that ring were being withdrawn upwards from the field produced by a constant current in the middle ring, and having a direction still shown by the arrows in the middle ring. Fleming's rule shows that such effect will be to generate in the upper ring an E.M.F. contrary in direction to that indicated in Fig. 20, and consequently similar in direction to that in the middle ring. Also the magnitude of such E.M.F. will be least when the rate of change of lines is least, and greatest when such change is greatest; that is, it will be least at the moment when the current in the middle ring is a maximum, and greatest when the current there is on the point of change from positive to negative.

A similar investigation will show that when the current in the middle ring is reversed in direction, we shall get an exactly similar state of affairs only in an opposite sense. And to some scale or other we can represent the relationship between the current in the middle ring and the E.M.F. set up in the top ring, as also that in the lower ring (which will obviously be of the same order), as is shown in Fig. 22, where C represents the value and direction of the current in the middle ring, and the dotted line marked ϵ represents the value (to some arbitrary scale) and direction of the E.M.F. induced in the other rings. In consequence of its opposing character, this E.M.F. is called a back E.M.F.; and just as it is induced in the upper and lower rings by the changing current in the middle ring, so and much more, therefore, will a similar E.M.F. be induced by the middle ring in itself—*self-induced*, and therefore called the—

B.E.M.F. of self-induction

Leaving for the present the determination of the exact value of such E.M.F., we have next to consider the conditions under which such a current as C in Fig. 22 can flow in a circuit having this magnetic property. It is quite clear that an

E.M.F. will be required at any instant freely acting in the circuit in such a direction as to do two things—

- (1) Nullify any B.E.M.F. which may be present; and
- (2) Leave sufficient over to send the current C against the ohmic resistance of R ohms.

This last quantity, RC_i , has been drawn in above C in Fig. 22, and all that remains to be done is to add, for all instants in time, RC_i to an amount equal and opposite to ϵ to

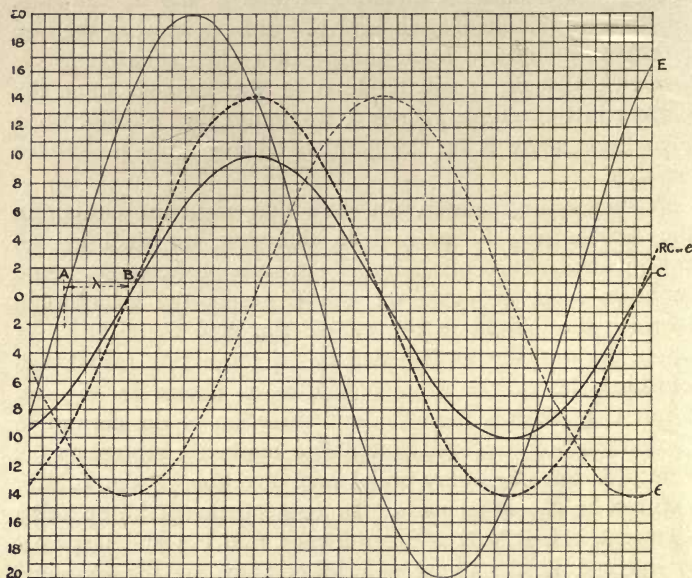


FIG. 22.

get the value of E , the E.M.F. which must be supplied to or impressed upon the circuit. This has been done after the manner indicated by Dr. Barfield in 1886,¹ and the curve E so produced is marked in full line. It will now be noticed that this E curve does not coincide in phase with the C curve which it produces; in fact, the C curve *lags* after E by an amount AB , which we will call the angle λ . A

¹ "Industries," vol. i. p. 17.

little consideration will show that this lag λ will depend upon the relative values of RC and ϵ , since E partakes most in character and phase with that one which is of greatest value. Thus, when ϵ is zero, as in a non-inductive circuit, $E = RC$, and there is no lag. But when RC is very small and ϵ very large, then E is equal and opposite to ϵ , and there is a lag of practically $\lambda = 90^\circ$.

Now, if we regard RC as a sine curve, it is clear that the opposite of ϵ must be a cosine curve; and as E is the sum of these two, we have that—

$$E = RC_m \sin \frac{2\pi}{\tau}t + \epsilon_{1m} \cos \frac{2\pi}{\tau}t$$

when RC_m is the maximum value of RC and ϵ_{1m} the maximum value of ϵ_1 . Now, this can be shown to be equivalent to—

$$E = \sqrt{(R^2C_m^2 + \epsilon_{1m}^2)} \sin \left(\frac{2\pi}{\tau}t + \lambda \right)$$

where λ is an angle whose tangent is—

$$\frac{\epsilon_m}{RC_m}$$

Calling $RC_m = e$ the active E.M.F., we may write shortly that the impressed E.M.F.—

$$E = \sqrt{e^2 + \epsilon^2} \text{ in magnitude}$$

and has a phase-position which is λ degrees earlier than the current, λ being such an angle that—

$$\tan \lambda = \frac{\epsilon}{e} = \frac{\text{B.E.M.F.}}{\text{active E.M.F.}}$$

Thus, when the circuit is of such a nature that a very powerful magnetic field can be set up by a small current, and the resistance is also small, there is a considerable lag of the current after the E.M.F., and such E.M.F. is very much greater than that necessary to merely cause the current to flow in the ohmic resistance of the circuit. But again, should the resistance of the circuit be very large, so that the active E.M.F. RC required is large compared to the B.E.M.F. set up by the

changing magnetism, then the current does not lag much, and E is only slightly larger than that required for RC.

This can also be graphically illustrated by remembering that the E.M.F. to be supplied, E , is the resultant of the other two E.M.F.'s which are present, viz. e and ϵ , which two are also at right angles. Thus we can construct a figure which shall show this resultant E.M.F., as is shown in Fig. 23, which shows the

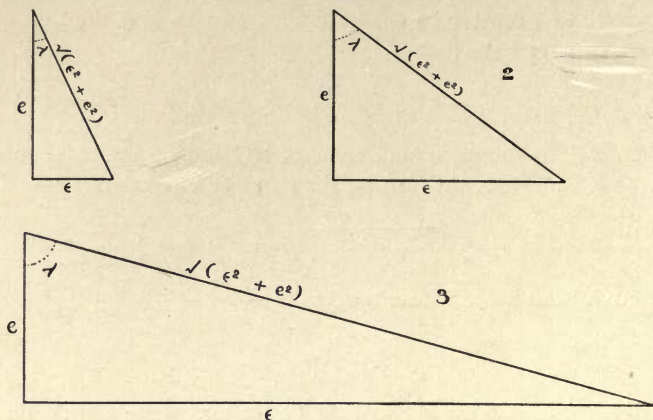


FIG. 23.

magnitude of the sum of e and ϵ , and also the nature of its position. Considering the phase-position of e to be represented by the vertical line, and remembering that the current C is in phase with e , it is clear from the three triangles in Fig. 23 that the difference in phase, as indicated by the departure from the vertical of line $E = \sqrt{e^2 + \epsilon^2}$, is least of all when ϵ is small compared with e , is very marked when ϵ is comparable to e in value, whilst, however large ϵ may become, the sum $E = \sqrt{(e^2 + \epsilon^2)}$ can never be more inclined to e than the horizontal position, or can never differ in phase from e by more than 90° .

Further investigation will also show that the rate of expenditure of energy in the circuit will depend upon the lag; being greatest when there is no lag, and least when the lag is large.

If the instantaneous values of the product of E and C be taken for the three possible cases, viz. no lag, small lag, and large lag, we shall get the waves of watts, as represented in Figs. 24, 25, 26, where the areas enclosed by the dotted lines

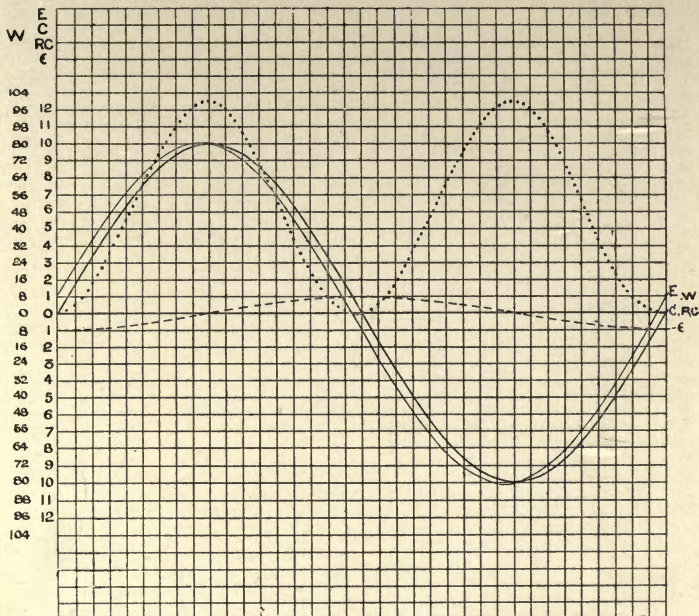


FIG. 24.

W will represent the work done. It will be noticed, in the case of no lag, that all the work is positive that is done in the circuit, and it can be shown that the average height of the watts area is then—

$$E_v C_v = \text{watts}$$

—the voltmeter reading multiplied by the ammeter reading.

But in the case of small lag there is part of the area of work marked on the lower part of the zero line and called negative; this must be understood as representing the case when the circuit is not receiving energy, but is giving it back to the engine, the resultant work being the difference between the +

and — areas, an amount which can be shown to be equal as an average value to—

$$E_v C_v \times \cos \lambda = \text{watts}$$

Lastly, in the case of maximum lag = 90° the + and —

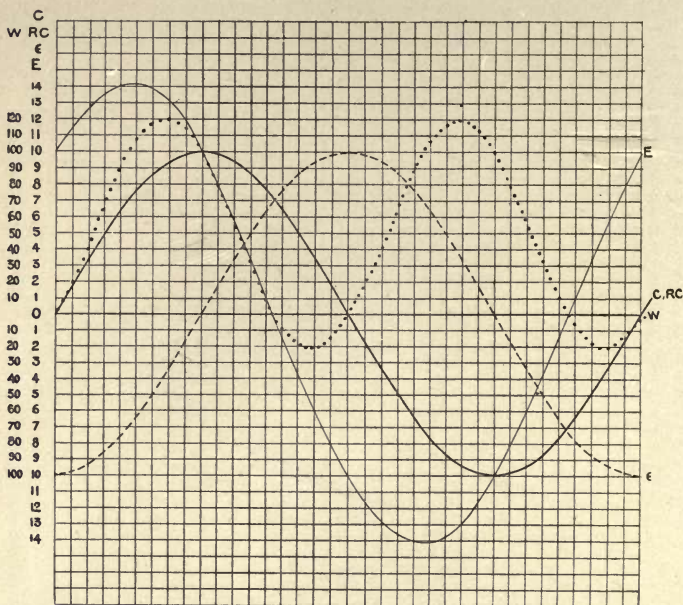


FIG. 25.

areas are equal, and consequently the work done nil. This can also be represented as—

$$\text{watts} = E_v C_v \times \cos \lambda$$

where $\lambda = 90^\circ$ and $\cos \lambda = 0$. Such a condition is, of course, not absolutely attainable, though in some cases the lag is very nearly 90° .

The coefficient $\cos \lambda$ has been called by Dr. Fleming the *power-factor*, and though it cannot be directly measured, it is of great importance. As shown above, its value can be found

from a knowledge of the ratio of the back E.M.F. to the active E.M.F., or of—

$$\tan \lambda = \frac{\epsilon}{e}, \text{ whence } \cos \lambda = \frac{e}{\sqrt{e^2 + \epsilon^2}} = \frac{e}{E}$$

The active E.M.F. is easily found for any circuit whose resist-

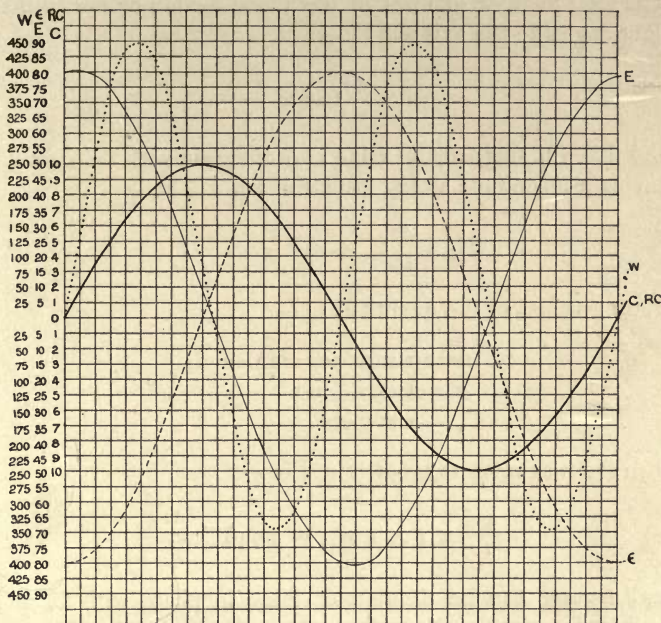


FIG. 26.

ance and current are known ; and we will now proceed to ascertain ϵ .

The magnitude of the B.E.M.F. set up in a circuit of constant $\mu = 1$.

Consider an infinite solenoid having no magnetic core, and being—

d cms. = mean diameter, *i.e.* d is the diameter to the middle of the depth of winding.

w = number of convolutions per cm. of length of coil.

The value of the magnetic density set up inside this solenoid will be—

$$\frac{4\pi}{10}Cw$$

when C is the instantaneous value of the current in ampères. The total number of lines in the plane section of the core at right angles to the axis will be—

$$N = \frac{4\pi}{10}Cw \times \frac{d^2\pi}{4}$$

and this will alternate in value and direction with the current, having a magnitude at any instant which will be—

$$N_i = \frac{4\pi}{10}C_m w \times \frac{d^2\pi}{4} \sin \frac{2\pi}{\tau}t$$

Putting values to the quantities of—

$$w = 1 \text{ turn per cm. length}$$

$$d = 2 \text{ cms., and}$$

$$C_m = 1 \text{ ampère}$$

then the maximum value of N will be—

$$\frac{4\pi^2}{10} \times 1 \times 1 \times 1 = 3.948 \text{ lines}$$

and a curve can be drawn coinciding in phase with C , and having a maximum value of 3.948, or—

$$N_i = 3.948 \sin \frac{2\pi}{\tau}t$$

Now, the E.M.F. which a changing magnetic field can set up is, in volts, equal to the number of lines gained or lost per second multiplied by the number of turns or convolutions so gaining or losing lines, divided by 10^8 . As before shown, the lines are changing at the greatest rate at the instant when they are zero—that is to say, the slope of the lines' curve is then a maximum.

Let the periodic time $\tau = \frac{1}{100}$ second, then the number of lines corresponding to the instant in time 0.000027 second after the zero value, will be—

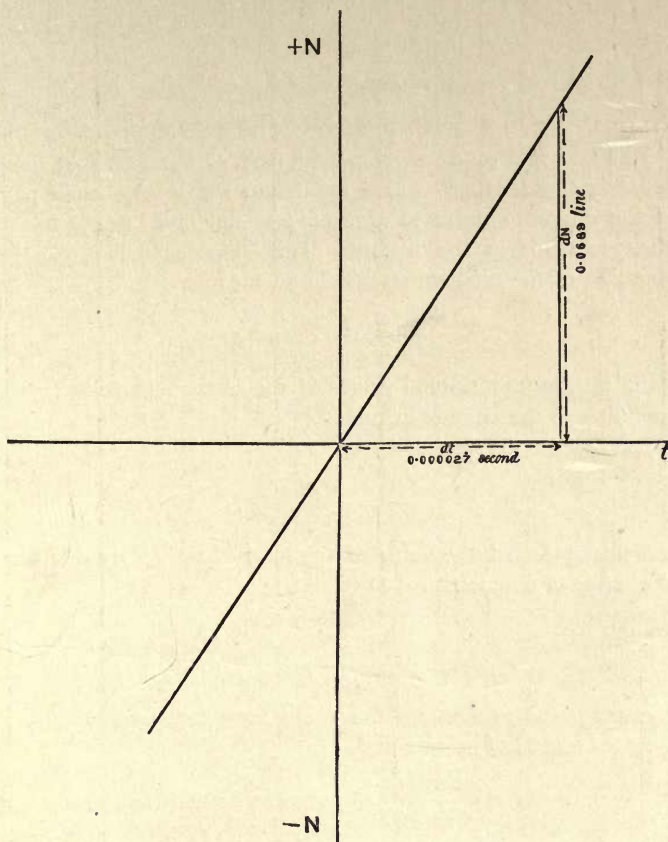


FIG. 27.

$$N_t = 3.948 \times \sin \left(\frac{2 \times 3.1416}{\frac{1}{100}} \times 0.000027 \right)$$

$$= 3.948 \sin 0.017452$$

which angle is in circular measure, and must be multiplied by $\frac{180}{\pi}$ to get degrees, or—

$$\begin{aligned} N_i &= 3.948 \sin \left(0.017452 \times \frac{180}{\pi} \right) \\ &= 3.948 \sin 1^\circ \end{aligned}$$

But $\sin 1^\circ$ is 0.01745, whence the value of N_i is—

$$N_i = 3.948 \times 0.01745 = 0.0689 \text{ line}$$

In Fig. 27 is given a magnified part of the curve of lines showing these values, which are taken for a very small part of the curve, in order to approximate the more nearly to the absolute slope at the instant. This slope is called, in the language of the differential calculus—

$$\frac{dN}{dt} \text{ when } dN \text{ and } dt$$

stand for the infinitesimal values of the change in lines corresponding to the change in time.

In our case the slope—

$$\frac{dN}{dt} = \frac{0.0689}{0.000027}$$

and we have that the value of the E.M.F. set up by this rate of change of magnetism is—

$$\epsilon = \frac{wdN}{dt} \text{ volts}$$

whence, putting values as above, we have for the E.M.F. per 1 cm. of length of infinite coil—

$$\epsilon = 1 \times \frac{0.0689}{0.000027} \div 10^8 = 0.0000248 \text{ volt}$$

which is the maximum value of the B.E.M.F. curve, and, as has been previously shown, has a negative value in accordance with Lenz's Law.

Now the value—

$$\frac{dN}{dt} = \frac{0.0689}{0.000027} = 2480$$

is the maximum rate of change of lines, and can be found from the maximum value of $N = 3.948$ by multiplying by $\frac{2\pi}{\tau}$ or 628.32 . Thus—

$$\frac{dN}{dt} \text{ maximum} = 3.948 \times 628.32 = 2480$$

and we can write the maximum value of the B.E.M.F. set up in a coil, subject to a magnetic field changing after the manner of a sine curve as—

$$\begin{aligned} \epsilon_{\max} &= \frac{2\pi N_{\max} w \frac{1}{\tau}}{10^8} \\ &= \frac{2\pi N_m w p}{10^8} \end{aligned}$$

putting $p = \frac{1}{\tau} =$ periods per second.

And the virtual value of this, as it is a sine curve, can be found by multiplying 0.7071 , or—

$$\begin{aligned} \epsilon_v &= \frac{0.7071 \times 2\pi N_m w p}{10^8} \\ &= \frac{4.443 N_m w p}{10^8} \text{ virtual volts per 1 cm. of length} \end{aligned}$$

In ascertaining the value of ϵ for any circuit, it is necessary to know N , and this is generally the real difficulty. So far we have considered an infinite solenoid; but in practice we have quite finite circuits to handle, and it is sometimes very difficult to accurately state their dimensions. If we next consider a solenoid of finite length, it will be at once perceived that the conditions are now no longer so simple, though we shall have the same value for the E.M.F., set up in the middle cm. length of the coil, provided the ends be fairly remote. Thus in a—

Finite Solenoid without magnetic core, we have that the B.E.M.F. per cm. length, at and near the middle is—

$$\epsilon = \frac{4.443 N_m w p}{10^8} \text{ volts virtual}$$

but near the ends, owing to the straying of lines, the value will not be so great. The coil will, in fact, act as though it had a length shorter than its actual length by an amount which is found to be very nearly equal to half the mean diameter. Thus in Fig. 28 is represented in section a coil having a total length

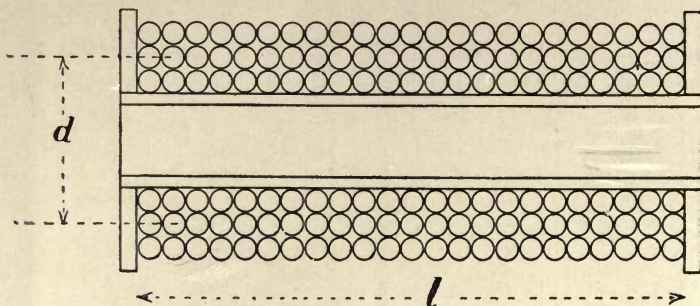


FIG. 28.

l cms., a mean diameter of d cms., and let us say of w turns per cm. length. If this coil were part of an infinite solenoid of similar construction, it would have a B.E.M.F. per cm. of its length of—

$$\epsilon = \frac{4.443 N_m w p}{10^8} \text{ volts}$$

or—

$$\epsilon = \frac{4.443 C_m w \frac{4\pi}{10} d^2 \frac{\pi}{4} w p}{10^8} \text{ volts virtual}$$

per cm. length, or a value of—

$$\epsilon = \frac{4.443 C_m w^2 \frac{4\pi^2}{40} d^2 p}{10^8} l \text{ volts}$$

total. But as it is finite, its B.E.M.F. will approximately be—

$$\epsilon = \frac{4.443 C_m w^2 \frac{\pi^2}{10} d^2 p}{10^8} \left(l - \frac{d}{2} \right) \text{ volts virtual}$$

which comes to saying that the effective number of lines set

up in this coil is not uniformly $C_m \frac{4\pi}{10} w$ per sq. cm. of cross-sectional area, but is only—

$$C_m \frac{4\pi}{10} w \times \frac{l - \frac{d}{2}}{l}$$

per sq. cm. average. Nevertheless the whole M.M.F. in ampère-turns is at the maximum crest—

$$\mathfrak{H} = C_m \frac{4\pi}{10} w l$$

whence it follows that part only of this is concerned in causing the flux through the cylindrical air core, the rest obviously being required to send the lines through the surrounding air space.

Now, the reluctance of the air core can be expressed as—

$$\mathfrak{K}_c = \frac{l}{a\mu} = \frac{l}{\frac{d^2\pi}{\mu}} = \frac{4l}{d^2\pi\mu}$$

and to send—

$$\frac{d^2\pi}{4} C_m \frac{4\pi}{10} w \frac{l - \frac{d}{2}}{l} \text{ lines}$$

through this reluctance will require a M.M.F. in ampère-turns of—

$$\frac{10}{4\pi} \times N \times \mathfrak{K}$$

or—

$$\frac{10}{4\pi} \times \frac{d^2\pi}{4} C_m \frac{4\pi}{10} w \frac{l - \frac{d}{2}}{l} \times \frac{4l}{d^2\pi} \text{ ampère-turns}$$

μ being unity; which may be simplified to—

$$C_m w \left(l - \frac{d}{2} \right) \text{ ampère-turns}$$

thus leaving $C_m w \frac{d}{2}$ ampère-turns to overcome the reluctance of

the air space surrounding the coil : and as the same number of lines, viz.—

$$\frac{d^2\pi}{4} C_m \frac{4\pi}{10} w \frac{l - \frac{d}{2}}{l} \text{ lines}$$

traverse this air space, it follows that the reluctance of it will be—

$$\begin{aligned} R_s &= \frac{\text{ampère-turns}}{0.8 \times \text{lines}} = \frac{C_m w \frac{d}{2}}{\frac{10}{4\pi} \times \frac{d^2\pi}{4} C_m \frac{4\pi}{10} w \frac{l - \frac{d}{2}}{l}} \\ &= \frac{1.273l}{d(2l - d)} = \text{reluctance of the air space} \end{aligned}$$

This expression is not often required in considering the behaviour of an air-core coil, but will be required when dealing with a composite core.

We have next to consider the effect produced when the medium surrounding the coil, *including the coil itself*, is more or less conducting.

(c) Air core, μ constant and unity, but medium conducting. These conditions will be fulfilled by the coils in Fig. 28, if some or all of them be individually joined so as to make complete electric circuits.

We have seen that a given impressed E.M.F.—

$$E = \sqrt{(e^2 + \epsilon_1^2)} = \sqrt{(R^2 C^2 + \epsilon_1^2)}$$

where ϵ_1 stands for the equal and opposite of the B.E.M.F., which is produced by the current C . It follows from this that the current which a given E.M.F. impressed on a circuit will produce must be sufficiently large to provide the necessary maximum number of lines to produce such a value of ϵ as will leave so much E.M.F. over as is required to send C ampères against the ohmic resistance R . In circuits as generally met with in transformers, impedance coils, and such like, the value of e is usually small compared to ϵ , and it is now necessary to

consider the effect which will be produced by variations in the magnitude of e for a given value of E . Thus, let—

$E = 100$ virtual volts

$e = CR$, and R be, say 1, whilst C for some cause or other is made to range from 1 to 20

We have, then, that—

$$\epsilon_1 = \sqrt{E^2 - e^2}$$

and will be as shown in the following table :—

| E | C | ϵ_1 |
|-----|----|--------------|
| 100 | 1 | 99.995 |
| 100 | 2 | 99.979 |
| 100 | 3 | 99.954 |
| 100 | 4 | 99.919 |
| 100 | 5 | 99.874 |
| 100 | 6 | 99.819 |
| 100 | 7 | 99.754 |
| 100 | 8 | 99.679 |
| 100 | 10 | 99.498 |
| 100 | 15 | 98.868 |
| 100 | 20 | 97.979 |

which shows that e may range from 1 per cent. to 10 per cent. of E , without affecting the value of ϵ_1 produced by more than $\frac{1}{2}$ per cent., and may even be 20 per cent. of E , with only a trifle more than a corresponding reduction in ϵ , of 2 per cent.

It consequently follows that the number of lines produced by an alternating P.D. applied to a coil will depend almost entirely upon the P.D., and will be almost independent of the nature of the core or medium surrounding the coil, even when that medium is of such a nature as to make the current required to magnetize so large as to represent a P.D. required for the ohmic resistance of as much as 20 per cent. of the impressed P.D. In other words, this comes to saying that the constancy of the magnetic field produced is, as it were, the sole concern of the impressed E.M.F. Should anything arise tending to undo the magnetic effect first set up, the result will be such an adjustment of the current with respect to the E.M.F. as to make the magnetic flux produced, and conse-

quently the B.E.M.F. still the same as at first, subject only to the restrictions indicated in the table on p. 135.

This important fact was first pointed out by the author in a course of lectures in the summer term at the Municipal Technical School, Manchester, in 1892, and was afterwards confirmed by some experiments mentioned by Prof. S. P. Thomson in a paper before the Physical Society in 1894.

Now, when the rings in Fig. 21 are considered to be closed, the E.M.F. set up in them can cause a current to flow, which current will obviously disturb the magnetic field set up by the middle coil unless, as indicated above, the current in that ring is altered in such a manner as to exactly balance such disturbance, and still leave sufficient to produce the original magnetic flux. Foucault currents or eddies set up in the mass of the copper coils, in the iron core, and in any other conducting body within the influence of the magnetic field are of this nature, as is also such a state as current flowing in one of the closed rings, which may thus represent a loaded secondary coil. To exactly balance the effect produced, there will have to flow in the middle ring a current which will be magnetically equal and opposite to these "secondary" currents. Such a current may be called the balancing current C_b , and when the current to be balanced does not lag after its own E.M.F. will come in opposite phase to the induced B.E.M.F., as is indicated in Fig. 29, where C_m stands for the current required to produce the magnetic flux before any "secondary" load came on, whilst C_b is such addition to C_m as will nullify the "secondary" load. The curve marked C_p is the sum of C_m and C_b , and may be taken as the current produced by the given E.M.F., acting in a circuit containing a given B.E.M.F. and secondary load.

The full consideration of such an effect will be deferred till the question of regulation in a transformer presents itself. Briefly we may sum up by saying that the effect of a conducting property in the system will only be to cause an alteration in magnitude and phase of the current, and affecting only in a very small degree the value of the magnetic disturbance set up.

And also that the number of lines produced in a given coil by a fixed alternating P.D. will be practically the same,

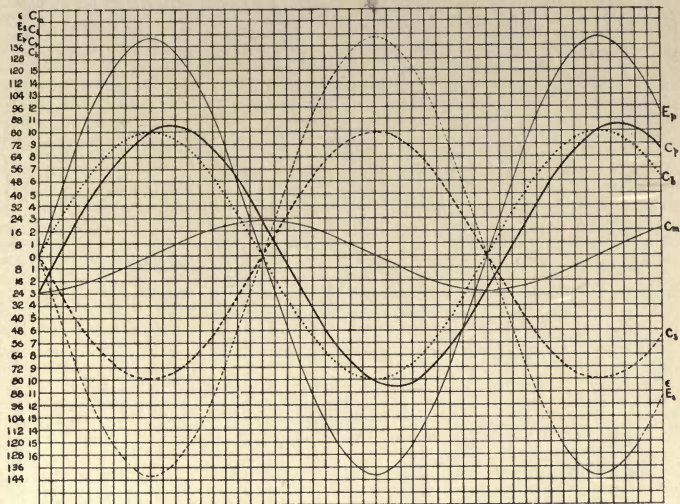


FIG. 29.

whether that coil has a core of air or of the best possible laminated iron.

(d) We have now to consider the case for a coil surrounded by a medium, such as iron, having a high but variable value of μ , and for simplicity so well laminated as to be non-conducting.

In iron of even the very best and softest quality it is found that the relationship between the magnetizing force and the flux produced is not of a simple nature; and further, that when the magnetizing force changes periodically, the flux changes periodically, but after a different law. In Fig. 30 is shown this relationship, the enclosed area being a measure of the work required to overcome the molecular friction in turning the particles of iron through a complete circle. Professor Ewing has published tests of a large number of samples of iron, and from his curves tables have been read off, showing the

values of H required for numbers of points round each cycle for different maximum values of β .

Corresponding with these a table is given, showing the waste

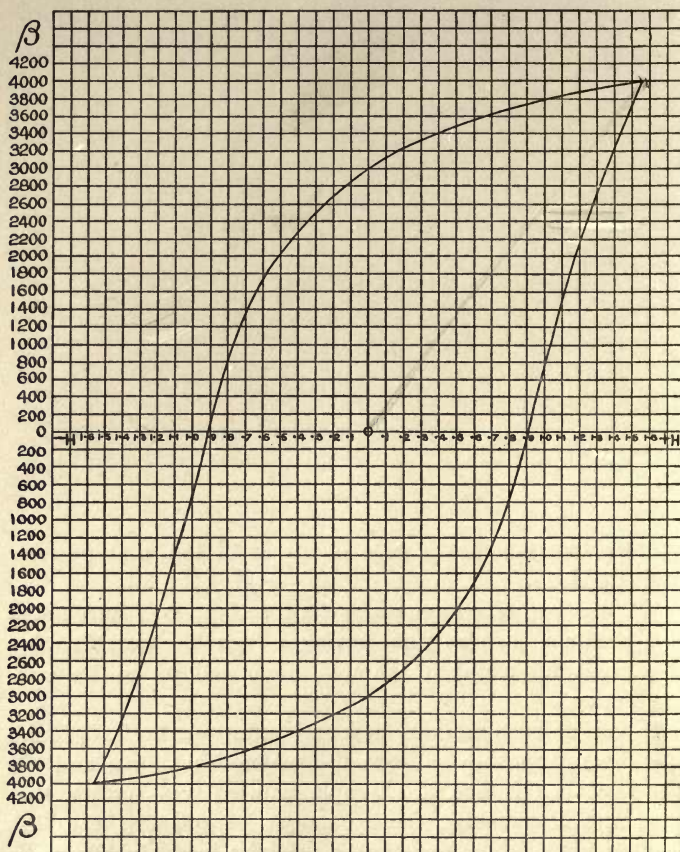


FIG. 30.

in friction in ergs per cubic centimetre per cycle, called h as before, for dynamo calculations.

Now, the nature of the motion of the iron particles will depend upon the change from moment to moment of the

E.M.F. acting in the circuit, and when this is of the simple sine curve nature, it is found that the value of β also changes after the manner of a sine curve. This is shown by the fact that the shape of the secondary curve of E.M.F. is practically a sine curve. Working backwards it is then easy, as was first pointed out by Mr. Evershed, to plot the corresponding values of current required when the curve of lines has been drawn. The cycle of magnetism in Fig. 30 is supposed to be that found when the iron has settled down to a steady state, and it will thus be seen that the manner of change is such as would be met with in going round the figure in a counter-clockwise direction. It thus becomes apparent that, owing to hysteresis, when β is zero, but becoming of positive magnitude, then C has to be of quite a considerable value, and does not increase nearly in accordance with the rise of β from 0 to maximum. Also that when the maximum β has been reached (which *always* coincides with the maximum value of the current), the current declines in value much more quickly than β , having even to become of opposite sign before β has again become zero. The reverse process takes place for the negative half wave of β , and a complete wave is shown in Fig. 31, where the current curve C is drawn to a larger scale than β for convenience. It will be noticed—

(1) That the value of C is very small, owing to the high value of μ , which is thousands of times what it would be in air.

(2) That the C curve does not coincide with β , though, of course, ϵ will have its customary position or lag 90° after β . Consequently, when the B.E.M.F. is the major item in determining E, E will still come in nearly opposite phase to ϵ , and consequently C will not lag so much after E as would have been the case with a similar air core.

This decreased lag is of great importance, as, together with the altered shape of the current curve, it determines the supply of energy which is wasted in hysteresis, in addition to that necessary for C^2R .

Now, we can easily draw in such a curve as that shown, and know its maximum value. But as indicated on an electro-dynamometer we shall of course get its virtual value, and this

will no longer be $0.7071 \times$ maximum value. Thus, in such cases as impedance coils made with a closed iron core, and where the current is made fairly large by making the value of

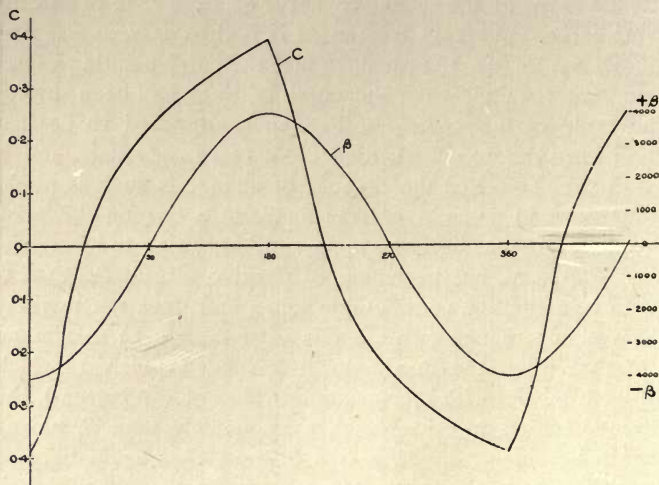


FIG. 31.

w small, this must be remembered. Owing mainly to the irregular shape of the C curve in such a circuit it is general to make impedance coils with an open iron circuit, the advantage of which will be indicated in the section devoted to the consideration of the open iron circuit.

(e) Consideration of the closed iron circuit when conducting, or having eddy current in the iron core, or a secondary load. The nature of the disturbance set up by Foucault currents, or a closed secondary will be of precisely the same character as in the other case considered; it is only necessary to point out that, as in general the magnitude of C_m , as we will call the current required to produce the flux in the iron, is very small, and lags little after E , whilst the current to balance the Foucault losses has a magnitude which is also small, but which is practically in phase with the impressed E.M.F., their sum is consequently larger than either, and lags less than C_m , whilst when there is a secondary load to balance there is practically

no lag of the current after E, for even a small value of the necessary balancing current.

(f) Medium composite in character, as, for instance, in the case of an open iron circuit impedance coil. Here, as in the composite circuit of a dynamo, calculation has to be made for the several items. Thus for the straight iron core of length l and sectional area a , to be magnetized to a total of N lines, we have with a given winding of coil surrounding it, that the M.M.F. required will be—

$$Cwl = lf\left(\frac{N}{a}\right) = lf(\beta)$$

a reference to the tables being necessary to find the values of C_w corresponding to various values of β .

In addition to this there will be required a current to send the same lines through the air space. This may be found by considering the reluctance of the air space, as given in the formula—

$$\mathfrak{K}_s = \frac{1.273l}{d(2l - d)}$$

where l and d now must be considered to have the values for the iron core. To force the lines through this space additional ampère-turns will be required of magnitude—

$$0.8N\mathfrak{K}_s$$

or—

$$C_1wl = 0.8N \frac{1.273l}{d(2l - d)}$$

Now, that part of the current C , which is required for the iron core, will be, as indicated, a saw-tooth curve like that in Fig. 31, will lag somewhat after E, but not nearly to 90° , and will also be small in magnitude compared to C_1 , which will be a sine curve, will be large, and will lag practically 90° after E. The total magnetizing current will consequently lag much, will partake mostly of the sine curve shape of C_1 and will, on the whole, be very large compared to the case of a closed iron circuit. An example will render this clear. Let the iron core

(laminated perfectly) be 30 cms. long and 5 cms. in diameter. Let the value of β be 5000 total, as an average of maximum value throughout the whole length. Let there be 4 turns per cm. length of core. Then—

$Cwl = lf(\beta) = C \times 4 \times 30 = 30 \times 1.41 = 0.35$ ampère maximum value of the saw-tooth shaped curve for the iron core. The total number of lines produced will be a maximum of—

$$N = 5000 \times \frac{5 \times 5 \times 22}{4 \times 7} = 98,200$$

And the reluctance of the air space will be—

$$\mathfrak{R} = \frac{1.273 \times 30}{5(60 - 5)} = \frac{1.273 \times 30}{5 \times 55} = 0.1388$$

Whence the ampère-turns required will be—

$$C_1wl = 0.8 \times 0.1388 \times 98,200$$

$$\text{and } C_1 = \frac{0.8 \times 0.1388 \times 98,200}{4 \times 30} = 90.86 \text{ ampères}$$

maximum value.

Now, the 0.35 ampères required for the iron core will lag after E considerably less than 90° ; but the 90.86 ampères, which is a sine curve, will lag 90° , and as its magnitude is so large compared to that for the iron, we shall get the real magnetizing current required, being practically a sine curve and lagging all but 90° after E, whilst its maximum value will only be slightly less than 91 ampères. Indeed, we may say that the iron core has nullified the reluctance of the core, and all we have to consider is the outer air space. Had there been no iron core the reluctance of the air core would have been—

$$\frac{l}{a\mu} = \frac{30}{19.65} = 1.526$$

and the current required would have been nearly 1000 ampères.

It is for this reason, amongst others, that open iron circuit impedance coils are generally used instead of those with a closed iron core. The advantages are—

- (1) Current curve is practically a sine curve.
- (2) Easy shape to wind and adjust.
- (3) Small quantity of iron required.

The Hedgehog transformer of Mr. Swinburne is an example designed to obtain a large all-day efficiency. This will be more fully dealt with in the section on transformers.

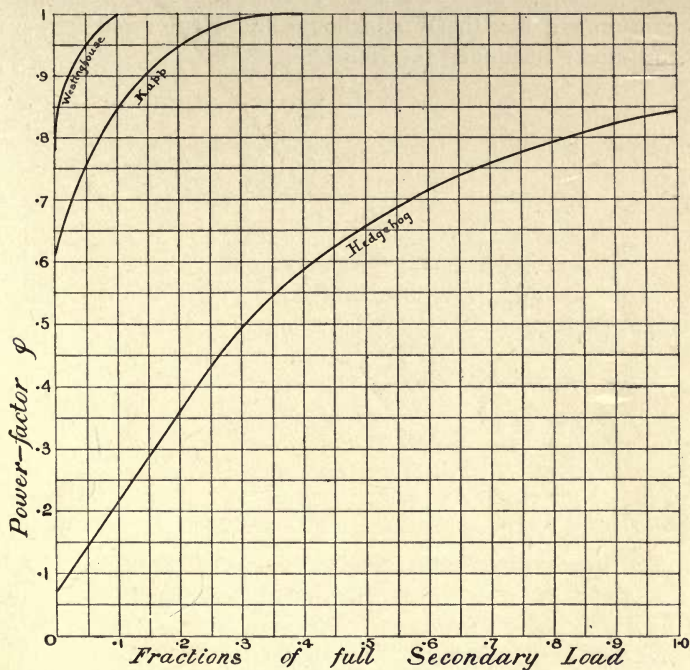


FIG. 32.

(g) When the medium of an open-iron circuit coil is conducting, as when imperfectly laminated or when containing a closed secondary circuit, we have the same balancing currents required. Only here such currents are no longer the only ones of any magnitude, and their effects in determining the total current are considerably reduced by the already large magnetizing current. It thus follows that even at very large

values of the secondary load there is still a very considerable lag of C after impressed E.M.F. In some curves, showing the relationship between the power factor and various loads for closed-iron and open-iron circuit transformers, published by Dr. Fleming in 1892, this great difference is very apparent. For convenience of reference they are here reproduced in Fig. 32, where the relationship is shown for two closed-iron transformers, viz. the Westinghouse and Kapp; and for one open-iron transformer, the Hedgehog.

CHAPTER XIII.

IMPEDANCE COILS AND TRANSFORMERS.

Impedance Coils, or choking coils, as they are sometimes called, are pieces of apparatus designed to make use of the property of self-induced E.M.F., by introducing such an E.M.F. into a circuit acting so as to decrease the active E.M.F. without at the same time absorbing any power except that incidental to the properties of the materials employed. Thus, we may have a pair of mains at an alternating P.D. of 100 (virtual) volts, and it is desired to connect to these a lamp taking a current of 10 ampères at a P.D. of 40 volts. With a continuous current circuit one way is to introduce a resistance in series with the lamp of 6 ohms, but then the energy wasted in this resistance will be at the rate of 600 watts. Or a number of secondary cells may be put in circuit, having together an E.M.F. acting against that of the mains of about 60 volts. In this case there will be a storage of the energy not required by the lamp, only a small part being wasted in heating, and other losses peculiar to secondary batteries. With the alternating current it is equally possible to introduce a dead resistance of 6 ohms, and waste the same 600 watts; but we can introduce a similar B.E.M.F., as in the case of the secondary battery, the problem being to know how much? Remembering, then, that this B.E.M.F. is to be produced by the passage of the current through an inductive circuit, we have that—

$$E = \sqrt{e^2 + \epsilon_1^2}$$

or—

$$e = \sqrt{E^2 - \epsilon_1^2}, \text{ or } \epsilon_1 = \sqrt{E^2 - e^2}$$

and inserting values in this last, we have—

$$\begin{aligned} e_1 &= \sqrt{(100 \times 100 - 40 \times 40)} \\ &= \sqrt{(8400)} = 91.65 \end{aligned}$$

This value is, however, slightly too large, since the very coil which is to produce it will have a resistance, and so absorb a small E.M.F. in becoming heated. In fact the impedance coil is also a resistance coil, and the effect of its own resistance should be allowed for as a resistance in series with the external circuit. This resistance of the coil is usually very small, compared to the resistance of the rest of the circuit, in order that the power wasted may be small.

Calling e_1 the active E.M.F. required by the coil itself, we then have—

$$e_1 = \sqrt{\{E^2 - (e + e_1)^2\}}$$

and all these values may be in either virtual or maximum values, as we please, provided they are all either the one or the other.

Transformers.—An unloaded transformer, that is with open secondary circuit, is simply a case of an impedance coil designed, as a rule, to take the minimum current and produce a B.E.M.F. differing from E the impressed volts by as small an amount as possible. In as far as, then, as it is unloaded, it can be dealt with in precisely the same manner as the impedance coils discussed above.

In allowing for the effects of eddy currents it has been pointed out that the disturbing effect produced by them had to be balanced (automatically) by the increase of the main current by an amount such as would produce ampère-turns magnetizing force sufficient for the purpose of overcoming the demagnetizing effect due to the eddy currents. A little consideration will show that the current taken from the secondary coil of a transformer will be of an exactly similar nature and will similarly have to be balanced by a corresponding increase of current in the primary. The consequence of this is that the current in the primary coil is not only variable in magnitude but is variable in phase.

As with impedance coils, there are two cases: the closed iron circuit and the open iron circuit.

Consider a simple type of closed iron circuit transformer wound so as to have the same number of turns on the primary, as on the secondary.

The current required on open secondary circuit will be of a kind represented in Fig. 31, including the two items—

- (1) True magnetizing current; and,
- (2) Balance for eddy currents.

The sum of these two will be a curve partaking most largely of the nature of the true magnetizing current, since the current to balance eddies is small and also lags due to self-induction in the core.

The phase position of this no-load current, which we will call C_0 , will be after the primary E.M.F. by an amount less than 90° on account of the hysteresis, but will be very small in magnitude.

Of the two items of C_0 the magnetic effect of the balance for eddy currents is nullified by the eddy currents, thus leaving only C_m , the true magnetizing current to be looked upon as causing so many ampère-turns to act upon the magnetic circuit, setting up an alternating magnetic field in the core which produces the E.M.F. in the secondary as an E.M.F. coinciding with the B.E.M.F. in the primary, and as we have already seen this will have a phase position approximately 180° after the primary E.M.F.

So far as the secondary is on open circuit, we get no effect due to its presence except in so far as that presence has probably increased the eddy current loss somewhat. But when the secondary is connected, as is most usual, to a non-inductive circuit, we get a current flowing in it which is in phase with its own E.M.F., but in contrary phase to the primary E.M.F. This current will, of course, mean so many ampère-turns acting upon the magnetic circuit, and these would produce a very serious disturbance were it not for their effect upon the momentary value of the B.E.M.F., causing the primary current to increase in magnitude and lag less. This increase is of exactly such a nature as would be produced by an additional

current in the primary acting in contrary phase to the secondary current, and of such a magnitude as to produce ampère-turns exactly balancing the magnetic disturbance set up by the secondary current.

The real new primary current will, of course, be the sum of this balancing current C_b , and the original no-load current C_o , which, it must be remembered, is itself the sum of two items previously stated. In the case of a transformer without iron core and with no eddy current waste, there will then be only two currents to consider, viz., the current C_m to set up the necessary magnetic flux, a current lagging almost 90° after the primary E.M.F.; and the balance for secondary load, a current which is in phase with the primary E.M.F., and thus 90° apart from C_m . Calling the latter C_b , we shall have that the current at any time in the primary coil will be—

$$C_p \times \sqrt{(C_b^2 + C_m^2)}$$

and its phase position will be after the primary E.M.F. by an angle whose tangent is approximately $\frac{C_m}{C_b}$, which shows that when C_b is large compared to C_m the lag of C_p after the primary E.M.F. may be almost nothing. In the case of the closed iron circuit type this is most marked; for even at as small a value of C_b as corresponds to only one-tenth of the full secondary load, we find the ratio of C_m to C_b so small as to make no appreciable lag of C_p after primary E.M.F.

But in the case of the no iron core or open iron circuit transformer, C_m is so large as to prevent the ratio of C_m to C_b ever becoming very small, even with a value of C_b corresponding to full secondary load. In the case of the open iron core we have C_p being composed of the sum of several items, viz. :—

(1) C_{mi} to magnetize the iron core: this is small in magnitude, is saw-toothed in shape, and lags less than 90° after the primary E.M.F.

(2) C_{ma} to magnetize the air space: this is large, is practically sinusoidal, and lags almost 90° after the primary E.M.F.

(3) C_b to balance the effects of secondary load and any

eddy current waste there may be in the whole apparatus: this is practically sinusoidal, and lags hardly at all after the primary E.M.F.

The sum of all these is mainly a sine curve, even when C_b is quite small; and its phase position is mainly determined by the relationship of C_{ma} to C_b . But as C_b never attains such a size compared to C_{ma} as to make—

$$\frac{C_{ma}}{C_b} = < 0.3, \text{ or so}$$

it follows that there is always considerable lag of C_p after the primary E.M.F.

This is strikingly shown in the curves reproduced from Dr. Fleming's paper on Transformers, on p. 143, Fig. 32.

In transformer design care must be taken that the loss of volts in resistance is very small; that the loss of power in hysteresis and eddy currents is small; and that, above all, these last are small, inasmuch as they are a constant load and so very materially affect the efficiency when taken over a period of time during which the load varies considerably.

The various losses may be thus stated—

(1) Drop of volts in the primary coil, causing the B.E.M.F. ϵ , which is a measure of the magnetic flux set up, to differ from E the impressed E.M.F. This loss is such as to cause the E.M.F.—producing power of the transformer to be smaller than E , by the amount $C_p R_p$ taken in its proper phase, and making the ratio of $\frac{\epsilon}{E}$ slightly less than unity. It is, in the closed iron circuit type, very small at no load, since, then, C_p is very small; and it only reaches a value of importance when the secondary is on full load. But in the open iron circuit type there is not nearly so large a variation in the value of C_p for various secondary loads, and so not as great an alteration in the primary loss of volts.

(2) The drop of volts due to ohmic resistance of the secondary, viz. $C_s R_s$, is always proportional to the secondary load in all types, and can be prearranged to have any value desired.

(3) A drop in the secondary due to the slightly inductive character of the winding. This will increase with the load being such as to make the effective E.M.F. in the secondary less than—

$$\epsilon_p \times \frac{w'_s}{w_p}$$

where w_s and w_p stand for the number of secondary and primary turns, in as far as the secondary coil acts in the same manner as an impedance coil in its own circuit. For this reason the number of windings w_s should be kept small.

(4) There is also a drop in E.M.F. on full load, due to leakage of magnetic lines. It has already been shown that a fully loaded transformer has two oppositely acting magnetizing forces in the circuit, and if these be not evenly distributed round the magnetic circuit the consequence is evident polarity, followed by inequality of the magnetic flux through the coils. In a good closed iron type with the primary and secondary coils well intermingled along the magnetic circuit this drop can be kept to as low as $\frac{1}{10}$ per cent.

(5) The loss due to eddy currents in the core; and, in the case of open iron circuit types, due to eddy currents in the winding.

Mr. Alexander Siemens has pointed out how this loss may be arrived at in the case of a core composed of fine iron wires. The following formula is adapted from his paper on the subject, and is the value of the watts lost in eddy currents per cubic centimetre of iron—

$$F = \frac{6 \cdot 6 p^2 \beta^2 d^2}{10^{12}} \text{ watts per cc. of iron}$$

where d stands for diameter of wire in cms.

p „ periods ω per second.

β „ c.g.s. lines per sq. cm. of iron section.

Mr. Evershed has also shown how to express the loss when the iron core is composed of thin iron plates, viz. :—

$$F = \frac{p^2 \beta^2 t^2}{10^{11}} \text{ watts per cc. of iron}$$

where t is the thickness of plate in cms.

(6) The hysteresis loss due to molecular friction in the iron. This may be very accurately found in the way shown for the armatures of dynamos by means of the table on p. 214. Thus—

$$H = \frac{hVp}{10^7} = \text{hysteresis loss in watts total}$$

Calling the total induction in the iron of a transformer N c.g.s. lines, and using the symbols already mentioned, we have, neglecting the slight difference at no load between ϵ_p and E_p —

$$Nwp = \frac{E_p \times 10^8}{4.443 \times p}$$

and $N = \beta a$, β being quite small, ranging from as low as 3000 to 6000, and seldom exceeding the latter, whence—

$$w_p a = \frac{E_p \times 10^8}{4.443 \times p \times \beta}$$

which gives an important product of the two unknown quantities in terms of known, or easily fixed values. Only experience can fix the relationship between the primary turns, and the sectional area of the core. On the one hand, if w_p be large we shall have, with a given weight of copper, a high resistance and large copper loss; but the iron losses H and F will be small. If, however, a be made large, the copper losses may be very small, but the iron losses will be large. The relationship between these two losses, copper and iron, will depend greatly upon the average output or load-factor. When the transformer is to be on full load always, then the highest efficiency will be attained when the copper losses are equal to the iron losses. But in general the conditions of supply render it impossible that the transformer can be on full load for more than a very small part of the 24 hours, whilst for a very large part it may be practically unloaded. The all-day efficiency can then be made higher by making the constant iron-losses smaller at the expense of large copper losses, which will only be of importance in themselves for the short times of high and full load. In this, of course, due regard must be had for a reasonable constancy in available secondary P.D.

Example.—A transformer for 40 ampères and 100 volts secondary output. In order that the efficiency at full load may be, say 95 per cent., it follows that the input must exceed the output by about 200 watts, which is to cover both the copper losses and the iron losses. If these be about equal, we must allow for something like 79 watts in hysteresis, from which, if the value of β be fixed, we can find the volume of iron. Thus from—

$$H = \frac{hV\dot{\phi}}{10^7}$$

we have—

$$V = \frac{H \times 10^7}{h\dot{\phi}}$$

and if β be 6000, $h = 1820$. Putting these values and 79 for H , we have, if $\dot{\phi} = 80$ W per second.

$$V = \frac{79 \times 10^7}{1820 \times 80} = 5425 \text{ cc.}$$

If the primary E.M.F. $E_p = 2000$ volts virtual, we have from—

$$\epsilon_p = \frac{4.443 N w_p \dot{\phi}}{10^8}$$

where N is the maximum induction, that—

$$N w_p = \frac{2000 \times 10^8}{4.443 \times 80} = 562,700,000$$

and as $\beta = 6000$, we can put—

$$w_p a = 93,800$$

where a is the sectional area of the iron core in sq. cms. Taking this to be 155 sq. cms., we have from the previously found volume that the average length of the iron core must be—

$$\frac{V}{a} = \frac{5425}{155} = 35 \text{ cms.}$$

If the core is composed of stampings of the shape and size

shown below, we shall require a pile of them 22·2 cms. of iron, not counting lamination.

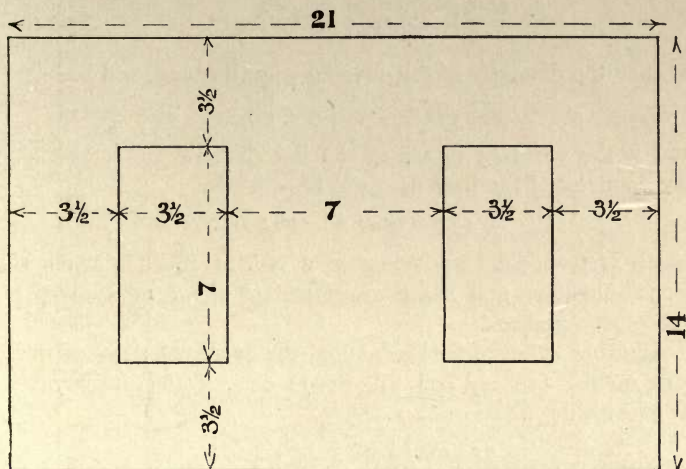


FIG. 33.

The total volume of iron in the above plates is—

$$(21 \times 14 - 7 \times 7) 22\cdot2 = 5439 \text{ cc.}$$

but as the magnetic induction cannot be taken as filling the iron into the corners, the amount under magnetic influence will be more nearly that previously found, viz. 5425 cc.

The sectional area is—

$$7 \times 22\cdot2 = 155\cdot4, \text{ say } 155 \text{ sq. cms.}$$

as was desired.

Of the spaces for winding we will allow that owing to superior insulation the space occupied by the primary coils will be 14·5 sq. cms. out of the total of 24·5 sq. cms., and that the secondary takes 8 sq. cms., leaving 2 sq. cms. for an insulating partition between the two coils. From—

$$w_p a = 93,800$$

putting $a = 155$, we have—

$$w_p = 606 \text{ say}$$

which number of wires has to go into the space of 14.5 sq. cms., or each wire will take up a room of—

$$\frac{14.5}{606} = 0.02392 \text{ sq. cm. area}$$

whence the diameter of the covered wire, if round, will be—

$$d = \sqrt{0.02392} = 0.154 \text{ cm. covered wire}$$

and if the covering increases the bar diameter by 0.07 cm., the diameter of the bare wire will be—

$$0.154 - 0.07 = 0.084 \text{ cm.}$$

which is practically equivalent to a No. 21 B.W.G., which is 0.032" diameter, and has a resistance of about 11 ohms per 1000 feet warm.

Allowing 2.8 cms. for lamination, the average length of one turn on the primary coil will be 72 cms. if the primary be wound inside the secondary.

$$72 \text{ cms.} = 28.5 \text{ inches say}$$

whence the total length of the primary wire will be—

$$606 \times 28.5 \text{ inches} = 17271$$

or 1440 feet. Whence the resistance—

$$R_p = 1.44 \times 11 = 15.4 \text{ ohms say}$$

Again, the mean length of the secondary turn will be 76 cms., or 30 inches, and—

$$w_s = \frac{1}{20} w_p = \frac{606}{20} = 31 \text{ say}$$

The space to be filled by these 31 turns is 8 sq. cms., whence each wire will occupy a room of—

$$\frac{8}{31} = 0.258 \text{ sq. cms.}$$

and if the wire be square the size will be $\sqrt{0.258} = 0.507$ cm. square covered, which will admit of a bare wire of 0.437 cm. side of square, about equivalent to B.W.G. $6\frac{1}{2}$, and having a sectional area of 0.1909 sq. cm., and a resistance of about 0.28^W per 1000 feet warm.

The total length of secondary wire will be—

$$\frac{30 \times 31}{12} = 78 \text{ feet say}$$

whence—

$$R_s = 0.28 \times 0.078 = 0.022 \text{ ohm.}$$

If, then, the maximum secondary current be 40 ampères, there will be a full load ohmic drop of—

$$40 \times 0.022 = 0.88 \text{ volts}$$

and a full load secondary copper loss of—

$$(40)^2 \times 0.022 = 35.2 \text{ watts, say } 36$$

Also, at full load the current in the primary coil will be—

$$C_p = \frac{C_s}{20} + C_m$$

where C_m is to include the magnetizing current giving the loss in hysteresis and also the current required to balance the eddy current loss in the core. In as far as these are losses they can easily be found, as shown below; but as they are very small compared to the secondary balancing current, they hardly at all affect the value of the primary copper loss. Thus, for example, corresponding to $\beta = 6000$ every centimetre length of the iron core will require 1.624 ampère turns, or the maximum magnetizing ampère turns on the primary will be—

$$35 \times 1.624 = 56.84$$

which must be produced by a current of—

$$\frac{56.84}{606} = 0.0937 \text{ ampères (maximum value)}$$

But this current is not a sine curve, and its virtual value will be greater than 0.707 times the maximum, or the virtual value will be, say 0.075 ampères.

Furthermore it is not in phase with the secondary balance, which has a virtual value at full load of—

$$\frac{40}{20} = 2 \text{ ampères}$$

Thus, including the hysteresis and eddy current losses, the

full load primary current cannot be greater than 2.1 amperes virtual, if so much, and therefore the primary full load copper loss is—

$$(2.1)^2 \times 14 = 62 \text{ watts say}$$

Using the formula on p. 150 for the eddy current loss, and taking the thickness of the stampings as 0.03 cms., we have the loss per cc. is—

$$\frac{80 \times 80 \times 6000 \times 6000 \times 0.03 \times 0.03}{10^{11}} = 0.0020$$

whence the total eddy current waste will be—

$$5425 \times 0.002 = 10.85 \text{ watts}$$

say 11 watts.

Thus, the total output will be 40×100 watts, and the input will be $40 \times 100 + 79 + 62 + 36 + 11 = 4188$, which is within the prescribed limit.

If kept on full load the efficiency will be—

$$100 \times \frac{4000}{4188} = 95.5 \text{ per cent.}$$

But when it is remembered that during a day of 24 hours a transformer on a lighting circuit is very variably loaded and may be practically on open secondary circuit for several hours, during which time the iron losses are constantly to be supplied, it will be apparent that the all-day efficiency or ratio of—

$$\frac{\text{Units paid for during 24 hours}}{\text{Units put in during 24 hours}}$$

will be a very different value to the full-load efficiency.

Consider, then, the case of the above transformer as on a circuit where it is—

| On | No load. | $\frac{1}{10}$ load. | $\frac{1}{4}$ load. | $\frac{1}{2}$ load. | $\frac{3}{4}$ load. | Full l ad. |
|-----|----------|----------------------|---------------------|---------------------|---------------------|------------|
| For | 8 hrs. | 4 hrs. | 4 hrs. | 4 hrs. | 3 hrs. | 1 hr. |

We shall have the following table of outputs and inputs for

24 hours, the total full load copper loss being 98 watts, a loss which will vary practically inversely as the square of the load; and the iron loss equal to 90 watts, whatever the load.

| Load. | Hours. | Output watt-hours. | Input = watt-hours. Output + iron loss + copper loss. | Total. |
|----------------|--------|-----------------------|--|--------|
| 0 | 8 | 0 | 0 + 8 × 90 + 0 = 720 | |
| $\frac{1}{10}$ | 4 | 4 × 400 = 1600 | 1600 + 4 × 90 + 4 × 0.98 = 1964 | |
| $\frac{1}{5}$ | 4 | 4 × 1000 = 4000 | 4000 + 4 × 90 + 4 × 6.1 = 4385 | |
| $\frac{1}{3}$ | 4 | 4 × 1334 = 5336 | 5336 + 4 × 90 + 4 × 10.8 = 5739 | |
| $\frac{1}{2}$ | 3 | 3 × 2000 = 6000 | 6000 + 3 × 90 + 3 × 25 = 6345 | |
| 1 | 1 | 1 × 4000 = 4000 | 4000 + 1 × 90 + 1 × 98 = 4188 | |
| Output 20936 | | | Input 23341 | |

And all-day efficiency = $\frac{100 \times 20936}{23341} = 89.6$ per cent.,

which is a very different thing to the 95.5 per cent. found above for the full-load efficiency, being even less than the efficiency if the transformer were kept on quarter load always, which would be over 91 per cent.

Example XLVIII.—A circuit has a B.E.M.F. of 1600 volts and a resistance of 20 ohms. What E.M.F. alternating will be required to send through it a current of 10 ampères?

Solution.—The relationship between the E.M.F. required E the B.E.M.F. ϵ and active E.M.F. $e = RC$, is that—

$$E = \sqrt{e^2 + \epsilon^2}$$

whence since—

$$e = RC = 20 \times 10 = 200$$

$$E = \sqrt{(1600)^2 + (200)^2} = 1612.45 \text{ volts}$$

Example XLIX.—In the above circuit what will be the current if the alternating E.M.F. supplied is 2000 volts, and what will be the rate of doing work?

Solution—

$$e = \sqrt{E^2 - \epsilon^2}$$

or—

$$e = \sqrt{(2000)^2 - (1600)^2} = 1200 \text{ volts}$$

but—

$$e = RC \therefore C = \frac{1200}{20} = 60^a$$

and the rate of doing work is $C^2R = 72,000$ watts or 96.48 HP.

Example L.—The P.D. at the terminals of a house connected to an alternating system of distribution is 100 volts. It is required to run 2 arc lamps, taking 34 volts each, in series off these terminals. If the lamps can be regarded as simple resistances of 3.4 ohms each, what will have to be the value of the B.E.M.F. of the choking coil to be used in series with them?

Solution.—Since each lamp takes 34 volts, and is equivalent to a resistance of 3.4 ohms, they are taking a current of 10 ampères, and when in series will require 68 volts, which is the value of R.C., the active E.M.F. required. Thus, since $E = 100$ we have—

$$\begin{aligned} e &= \sqrt{E^2 - e^2} = \sqrt{((100)^2 - (68)^2)} \\ &= \sqrt{(10000 - 4624)} = \sqrt{(5376)} \\ &= 73.32 \text{ volts B.E.M.F.} \end{aligned}$$

Example LI.—In the last question, what is the saving effected, by using the choking coil instead of a non-inductive resistance, if the choking coil consumes energy at the rate of 30 watts?

Solution.—The total expenditure with a non-inductive resistance would be $CE = 1000$ watts, and with the choking coil it is only—

$$C^2R + 30$$

or—

$$100 \times 6.8 + 30 = 710 \text{ watts}$$

whence the saving is—

$$1000 - 710 = 290 \text{ watts}$$

Example LII.—What will be the angle of lag of the current in Example L. ?

Solution, the lag is such that $\tan \lambda = \frac{\epsilon}{e}$

or—

$$\tan \lambda = \frac{73.32}{68} = 1.078$$

whence $\lambda = 47^\circ$ about.

CHAPTER XIV.

EFFECTS OF CAPACITY.

THE properties of a condenser may be easily studied by reference to the water analogy suggested by Dr. Fleming, as a mechanical representation of the electrical conditions.

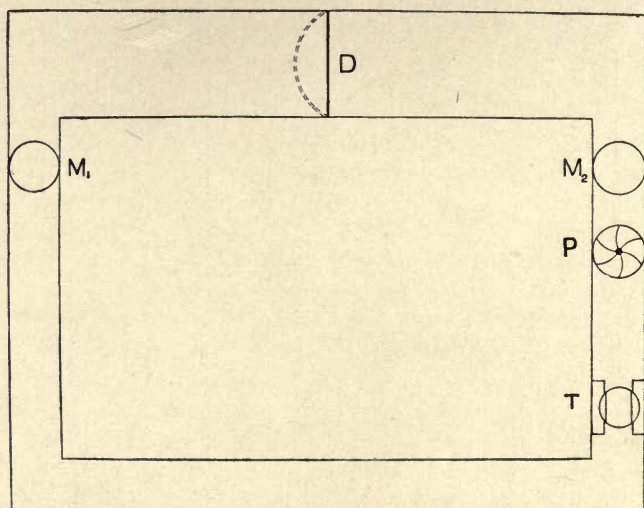


FIG. 34.

In Fig. 34 is represented a system of tubes containing water, divided at D by a flexible diaphragm of indiarubber, and having in each leg a water-meter, M_1 and M_2 . A tap and pump are also placed in one leg; the tap to represent a switch

in an electric circuit, and the pump corresponding to the electro-motive force.

First of all suppose the diaphragm removed. On working the pump a current of water will flow round the circuit in one direction or another, according to the rotation of the pump. This current will be indicated on the water-meters, and will be at the rate of so many gallons per minute—corresponding to an electrical effect of so many coulombs per second. This flow of water is constant in direction and in quantity, so long as the pump be rotated in one direction at the same rate.

If, however, the pump be replaced by a simple piston reciprocating in one of the tubes, we shall have the current of water constantly varying in direction, according as the piston is moving up or down; this reciprocating motion of the piston corresponds to an alternating E.M.F., and the alternating flow produced by it will not make any permanent record on the meters, since as much flows through in one sense as flows back in the other sense. The force urging the piston on its course will vary from point to point of the stroke, according as the mass of water is moving with or against it, and according to the resistance to the flow due to friction against the sides of the tube. This friction corresponds to resistance and the momentum of the water to the B.E.M.F. of self-induction.

But though the meters record nothing, they may be replaced by suitable mechanism indicating the amplitude of the reciprocating motion, which is a measure of the work-doing property of the moving mass of water.

When, however, the diaphragm is replaced we have a very different state of affairs. Formerly there was no obstruction to the motion of the water right round the circuit; but now, on rotating the pump, water will, let us say, be forced up the right side, causing the elastic diaphragm to swell out, as shown by the dotted line in the figure. Meter No. 2 will record the quantity of water required to do this as a quantity of water flowing *into* the diaphragm; but meter No. 1 will also record it as a quantity of water flowing *out* of the diaphragm. For however long the pump be worked the diaphragm will only become distended to an extent equal to the force exerted by

the pump, supposing, of course, that its strength is such as not to admit of rupture by any force the pump can exert. Thus, there will be a flux of water on the one hand into the diaphragm, and on the other hand out of the diaphragm for a short time only; and the rate of flow will have been larger at the start when the elastic force exerted by the diaphragm was small, than at the close when the elastic force due to stretching is equal, and opposite to the force exerted by the pump, thus leaving no available force for urging water along the circuit.

Whilst still keeping the pump in rotation if the tap be closed, the stretched condition of the diaphragm may be preserved indefinitely, although the pump be afterwards stopped.

A careful examination will disclose the fact that the only difference now existing in the apparatus is this stretched condition of the diaphragm: not a store of water, for as much obviously flowed in as flowed out, but a power to do work by relaxing—a store of energy.

The above conditions correspond to that of the continuous-current circuit, and the flux of water which lasts for a short time only after the starting of the pump is called the “displacement current.”

When the pump is now replaced by a reciprocating piston, we have a representation of the alternating-current circuit. Now, the diaphragm will be stretched first in one direction and then in the other, and the water-meters will, as before, be affected by a quantity of water surging to and fro in the tubes. The rate at which this quantity of water will be flowing in any one direction will depend upon the force of the piston and upon the elastic force of the diaphragm. When unstretched, the water will flow into the stretching diaphragm at a great rate, even though the force due to the piston may be small; but as the diaphragm becomes distended, the rate of flow into it will be small, and ultimately reach a zero value as the elastic force of the diaphragm equals that of the piston at the end of its stroke. As the pressure on the piston which caused it to move forwards is relaxed, allowing it to return to its starting position, the elastic force of the diaphragm causes water to flow

back, slowly at first, but reaching a maximum rate as the piston reaches its starting-point (the middle of its total stroke), and then again declining to zero as the piston moves on its opposite stroke, the diaphragm now becoming distended in the opposite way. This can better be seen by reference to Fig. 35, where P is the reciprocating piston, D the diaphragm,

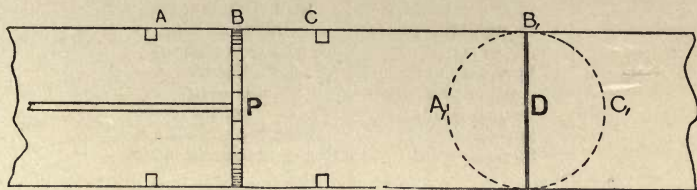


FIG. 35.

and ABC represent the course of the piston. As it starts from B the diaphragm is unstretched as at B_1 , and the rate at which water will flow into it is greatest; as the piston reaches C the diaphragm is distended to C_1 , and the rate of flow is a minimum, having declined from a maximum, but being always in the same direction as the force applied to the piston. As the piston, displaced thus far by a force, moves back to B by a removal of that force, the water now flows out or back towards the piston, not away from it as heretofore, till the piston reaches B, when the diaphragm is now as at B_1 the rate of flow, however, being now a maximum, but declining to zero again, though still towards the piston, as the piston moves towards A, due to a reverse force to that which urged it from B to C. The diaphragm is now as at A_1 exerting a force away from the piston, and as the latter returns, due to the gradual reduction of the outside force acting on it, the water now flows away from the piston at a slow rate at first, but reaching the maximum again as the piston returns to B, when the diaphragm is once more unstretched.

Calling the outside force which urges the piston along as equivalent to the alternating E.M.F. E, and the current of water which flows into the diaphragm as the current C, there

being no resistance to the flow and no magnetic effect set up, we thus have the following order of effects—

| E.M.F. — E. | Current — C. |
|---|--|
| Zero but rising to a + maximum then declining though still + to zero reversing and rising to a — maximum and declining to zero again. | A + maximum but declining to zero then reversing and rising again to a — maximum declining whilst still — to zero reversing and rising to a + maximum again. |

and so on, which can best be indicated by plotting the curves of E.M.F. and current on the assumption that the variations of strength of E.M.F. are sinusoidal, as is done in Fig. 36, where E is the E.M.F. supplied and C the current, the circuit

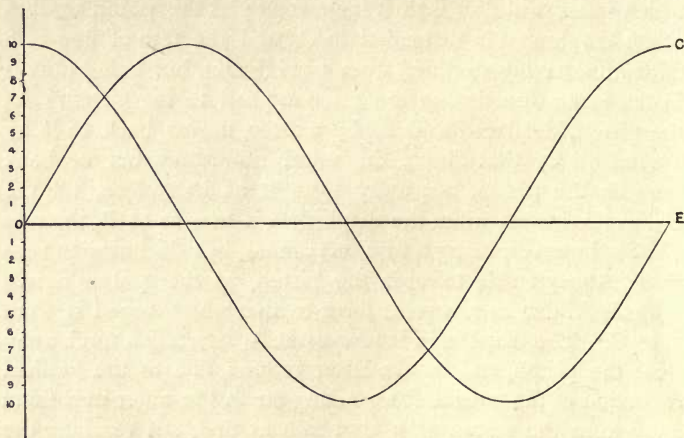


FIG. 36.

being supposed to be unable to set up any magnetic effect and to offer no resistance to the passage of the current, a condition of affairs which would correspond, on the water analogy, to a massless fluid moving in frictionless tubes.

Now, the degree to which the diaphragm can be distended by any force will depend upon its size and upon the material of which it is made; or its capacity depends upon its construction. Similarly the electric condenser has a capacity which is proportional to its area, inversely proportional to its dielectric thickness and also proportional to the property of the material of which it is made.

Thus the quantity of water required to distend the diaphragm so that its elastic force is equal and opposite to the unit force acting on the piston will be a measure of the capacity of the system. Also, in the electric circuit we may say that the quantity of electricity in coulombs required to strain the dielectric to an extent equal and opposite to the E.M.F. of 1 volt is a measure of the capacity of the condenser; and when that quantity is unity (1 coulomb) the condenser is considered to have unit capacity, or a capacity of 1 farad. Thus—

1 farad is that capacity which requires 1 coulomb to produce a P.D. of 1 volt between its two coatings.

But this quantity Q coulombs must, in an alternating-current circuit, be passed into, or out of, the condenser each quarter period, and therefore the average current flowing into, or out of, the condenser will be—

$$C_a = \frac{Q}{\frac{\tau}{4}} = \frac{KE_m}{\frac{\tau}{4}}$$

where K is the capacity in farads and E_m is the maximum value of the E.M.F. supplied to the condenser.

Or, putting $\phi = \frac{1}{\tau}$, we have the average current—

$$C_a = 4\phi KE_m$$

and thus the virtual current and maximum current will be—

$$C_v = 4.44\phi KE_m$$

$$\text{and } C_m = 2\pi\phi KE_m$$

But if E be stated as the virtual E.M.F., then—

$$C_v = 2\pi\phi KE_v$$

And we have seen that the E.M.F. supplied to the condenser reaches its maximum value, 90 degrees, later than the maximum value of the current due to it, the condenser itself exerting an E.M.F. in the circuit, always equal and opposite to the supplied E.M.F. It is necessary to make a very careful distinction between these two E.M.F.s; that supplied to charge the condenser is of course that of the dynamo, or other generator, and is called, as before, the impressed E.M.F.; that exerted by the condenser is of the nature of a back E.M.F., and may be called the B.E.M.F. of the condenser, κ .

Thus, if a condenser be in a circuit having an E.M.F. of E_v volts, it will take a current—

$$C_v = 2\pi p K E_v \text{ ampères}$$

and will exert a B.E.M.F. of—

$$\kappa_v = \frac{C_v}{2\pi p K} \text{ volts}$$

With a perfect condenser having plates of some material offering no resistance to the flow of the charge into them, and with a dielectric of perfect insulating properties, this B.E.M.F. κ_v will be exactly equal and opposite to E_v , the impressed E.M.F. But since some E.M.F. is required to maintain the momentary charging current, and there is a small actual leakage from one set of plates to the other (in both cases causing heating), it follows that the impressed E.M.F. E_v must slightly exceed the B.E.M.F. κ_v , the difference of phase being also slightly less than 90 degrees.

Now, since the supplied E.M.F. lags 90° after the current in the condenser, and the condenser B.E.M.F. is equal and opposite, it follows that the current in a condenser lags 90° after its own B.E.M.F. Therefore the effect of capacity in a circuit is to oppose the effect of self-induction, as may be easily seen by reference to the following diagram (Fig. 37), where the current through a circuit is marked as C ampères, the circuit being able to set up a magnetic field of such magnitude as to cause a B.E.M.F. of self-induction of ϵ volts, and also possessing a capacity such as will give κ volts, which E.M.F. may now be

called a forward E.M.F., in contradistinction to the B.E.M.F. of self-induction ϵ . To maintain the current C ampères through

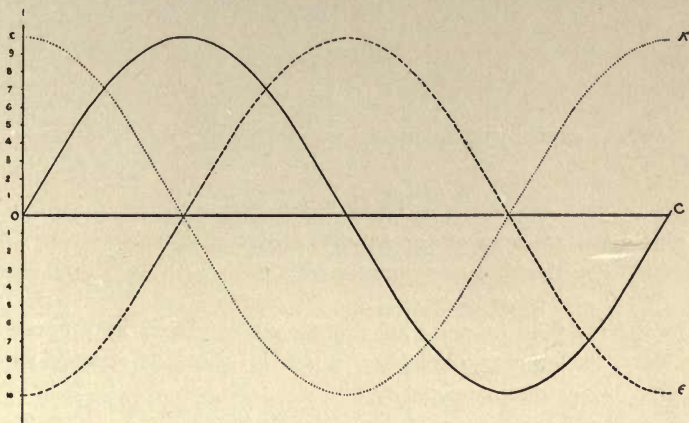


FIG. 37.

the circuit, the impressed E.M.F. E will thus have to be sufficient at any moment to (a) send C ampères through R ohms, (b) to nullify ϵ , and (c) to nullify κ ; and when $\epsilon = \kappa$, it follows that $E = e = RC$ simply, or the capacity effect has counteracted the magnetic effect.

Now, we have seen that—

$$\epsilon_v = \frac{4 \cdot 44 N_m w p}{10^8} = \frac{2 \pi N_v w p}{10^8} \quad (\text{see p. 132})$$

and also that—

$$\kappa_v = \frac{C_v}{2 \pi p K} = \frac{C_m}{4 \cdot 4 p K}$$

whence it follows, that when the capacity is such as to make the circuit non-inductive in spite of a magnetic effect—

$$\frac{2 \pi N_v w p}{10^8} = \frac{C_v}{2 \pi p K}$$

whence—

$$K = \frac{C_v 10^8}{4 \pi^2 p^2 N_v w}$$

or if N and C be maximum values, then—

$$K = \frac{C_m 10^8}{N_m \pi d} \times \frac{1}{4\pi^2 p^2}$$

or—

$$K = \frac{C_r}{\epsilon_r} \times \frac{1}{2\pi p}$$

Now, the capacity of a condenser in farads is—

$$K = i \frac{A}{1.131 \times 10^{13} \times d}$$

where A = the area of one set of coatings in sq. cms.

d = the distance between one set of coatings and the other in cms.

and i = that property of the dielectric placed in between the two coatings, which is called its specific inductive capacity.

The values of i for different materials are :—

| | | | | | | |
|-----------------------------|-----|-----|-----|-----|-----|----------------|
| Air | ... | ... | ... | ... | ... | 1 |
| Mica | ... | ... | ... | ... | ... | 5 |
| Flint Glass | ... | ... | ... | ... | ... | 6.5 to 13 |
| Glass | ... | ... | ... | ... | ... | 3 |
| Ebonite | ... | ... | ... | ... | ... | 2.5 to 3.1 |
| Sulphur | ... | ... | ... | ... | ... | 2.8 to 3.8 |
| Paraffin wax | ... | ... | ... | ... | ... | about 2 |
| Paper baked in paraffin oil | ... | ... | ... | ... | ... | about 2.7 to 3 |

In constructing a condenser, the conductors are composed of thin sheets of metal, usually tinfoil, piled upon each other with some dielectric in between. In order to obtain the large surface required for even a small capacity, the sheets are made of a reasonable size, and interleaved with each other, as shown in section in Fig. 38, where the lines show the metal sheets, and the spaces the dielectric. It will be noticed that five alternate plates are connected together by the terminal block T_1 , and are interleaved with the other six, these last being also all connected together by the terminal block T_2 . In such an arrangement use is made of each side of each plate, and thus the surface of such a condenser is found by the products of half the number plates into the surface of

one plate counting both sides. If, consequently, each plate measures 24 cms. by 48 cms., the area per plate will be 2304

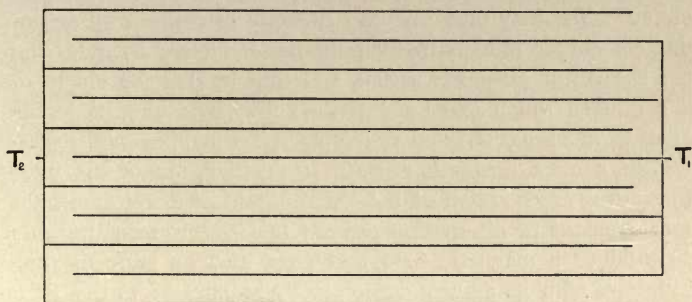


FIG. 38.

sq. cms. If now the dielectric be of oiled paper, 0.015 cm. thick, each piece of paper and plate of tinfoil will be, say 0.017 thick, or there will be $\frac{100}{0.017} = 5882$ plates, or 2940 say connected to one terminal per metre thick, and taking the specific inductive capacity as 3, the capacity per metre thick will be—

$$K = 3 \times \frac{2940 \times 2304}{1.131 \times 10^{13} \times 0.015} = 0.000119 \text{ farad}$$

or to make a condenser of 1 farad capacity would take a thickness of about 8400 metres, which gives some idea of the great size of a condenser to have so large a capacity.

Also, if used on a circuit working at 2000 virtual volts at 100 ω per second, the current taken will be—

$$\begin{aligned} C_v &= 2\pi\rho KE_v \\ &= 628 \times 1 \times 2000 = 1,256,000 \text{ ampères} \end{aligned}$$

to which should be added such current as may leak from one set of plates to the other, a current which will probably be almost in phase with the E.M.F. supplied, and consequently lagging 90° after the condenser current.

Use of Condensers for modifying the effect of self-induc-

tion. We have already seen that the nature of the B.E.M.F., set up by a condenser, enables it to oppose the E.M.F. of self-induction when the condenser is placed in series with the circuit. We may thus say that capacity in series with an inductive circuit renders that circuit non-inductive by nullifying the E.M.F. of magnetic action, and thus leaving the circuit in a condition which can be described fully by saying its resistance is R ohms, it requires simply RC volts, but can, nevertheless, exert a magnetic effect. In practice, one of the most interesting cases is that of the shunt circuit of a watt-meter for measurement of alternating power; this circuit must be able to produce a magnetic field, and must also be perfectly non-inductive with its current consequently in phase with the P.D. at its terminals.

Condensers are, however, sometimes used in parallel with a circuit the properties of which they are desired to modify. The circuit to which a condenser is a shunt is, however, unaltered in any way, the main effect is in the generator and leads connecting it to the circuit and its shunting condenser. Thus, if we have a circuit such as the primary of an unloaded open-iron circuit transformer, we have seen that the current taken by it will be large, but will lag almost 90° after the E.M.F. supplying it. This current will not necessarily mean any great waste of energy in the transformer, but the loss in the connecting leads and dynamo armature will be a considerable item. If now a condenser be connected across the primary terminals, the dynamo will have to supply through the leads a current such as will be determined by the size of the condenser, but coming 90° earlier than the E.M.F., and consequently opposed to the transformer current. The current, therefore, which flows in the leads and armature will be the sum of these two, and may be almost zero when the condenser current is equal to the transformer current, being only so large as will, in combination with the E.M.F., represent the waste in the transformer and condenser.

We have seen that the value of the current flowing into a condenser is—

$$C_r = 2\pi p K E_r$$

whence in this case we must make the capacity such as will take this current C_v equal to that required by the unloaded transformer at a P.D. of E_v volts, or—

$$K = \frac{C_v}{E_v} \times \frac{1}{2\pi p}$$

where C_v stands for the so-called “magnetizing” current of the open-iron circuit transformer.

The use of condensers for the above purpose was proposed by Mr. Swinburne, in conjunction with his “hedgehog” transformer, but very few are in use.

It is clear from the above that a condenser in parallel with a circuit tends to make the whole combination non-inductive, rather by rendering the apparent resistance larger, than by opposing any B.E.M.F. in the circuit.

In systems using concentric cables, the capacity of the cable considered as two conducting masses intimately associated but separated by the insulating covering on the inner conductor, is a shunt to the circuit supplied by those leads, and so may modify the effect of self-induction in the circuit. Messrs. Cowan and Still have published the values of the capacity of a few cables, viz.—

For $\frac{1}{18}$ concentric cables the capacity per mile is—

| | | | |
|---|-----|-------|------------|
| British Insulated Wire Co. (paper insulation) | ... | 0.31 | microfarad |
| Glover and Co. (vulcanized rubber) | ... | 0.615 | „ |
| „ „ (diatrine) | ... | 0.315 | „ |

This capacity will take a current coming 90° earlier than the impressed E.M.F., which current must be added to the current supplied through the cable to the load at the far end. When the load current is lagging much, due to self-induction, the total current will be, as shown above, the difference between the two, and therefore may be zero.

Example LIII.—A circuit supplied from an alternator takes a current of 10 ampères at a pressure of 100 volts, its resistance being, however, only 5 ohms. Find the lag of the current and the capacity in microfarads of the condenser which must be used in series with the circuit in order that the current may be

20 ampères, and not lag at all. There are 100 periods per second.

Solution.—The active E.M.F. will be 50 volts, whence the B.E.M.F. ϵ is $\sqrt{10000 - 2500} = 86.6$ volts, whence there will be a lag of current after the E.M.F. such that $\tan \lambda = \frac{86.6}{50}$ or $\lambda = 60^\circ$. In order to reduce this lag to zero, and thus make the current 20 ampères, the capacity will have to be—

$$K = \frac{20}{86.6} \times \frac{1}{2\pi 100} = 0.000367 \text{ farad}$$

or 367 microfarads, a capacity which could only be made in a size of about 25 cubic feet.

Example LIV.—An unloaded open-iron circuit transformer is shunted by a condenser of 30 microfarads capacity, and appears to take a negligibly small current. It is supplied at 2000 volts off a circuit, having 80 periods per second. What is the true magnetizing current?

Solution.—It is clear that the condenser current is equal to the magnetizing current as, being in opposite phase, it has just nullified it. Now, the current taken by such a condenser will be—

$$2\pi pKE = \frac{44}{7} \times 80 \times 0.00003 \times 2000 = 30.17 \text{ ampères}$$

which is, therefore, the true magnetizing current.

Example LV.—A concentric main six miles long, having a capacity of 0.7 microfarad per mile, is supplying energy to an inductive load, taking 10 ampères and lagging almost 90° . If the P.D. is 1000 volts, what is the indicated current where there are 100 ω per second?

Solution.—The current taken by the main on account of capacity will be—

$$C = 2\pi pKE = \frac{44}{7} \times 100 \times 4.2 \times 10^{-6} \times 1000 = 2.64 \text{ ampères}$$

which current will be almost in opposite phase to that in the load, whence the apparent current will be the difference or—

$$10 - 2.64 = 7.36 \text{ ampères}$$

Example LVI.—A concentric cable is 12 miles long, and is connected to an alternating P.D. of 10,000 volts. If the capacity is 1.5 microfarad per mile, what will be the current supplied, the circuit working at 80 ω per second?

Solution.—Total capacity is $12 \times 1.5 = 18$ microfarads, whence the current taken is—

$$C = 2\pi KpE = \frac{44}{7} \times 0.000018 \times 80 \times 10,000 = 90.5 \text{ ampères}$$

Example LVII.—Two condensers of $\frac{1}{3}$ microfarad capacity each are connected to an alternating P.D. of 50 volts at 225 ω per second. What will be the current taken—

(a) when the two condensers are in parallel?

(b) when in series?

Solution.—In parallel the total capacity will be $\frac{2}{3}$ microfarad, since virtually the surface of the condenser is now double that of either. Thus the current will be—

$$\frac{44}{7} \times 0.00000667 \times 50 \times 225 = 0.047 \text{ ampère}$$

But when in series the effect is the same as making the thickness of the dielectric twice as much while the surface remains the same. The capacity is then only $\frac{1}{6}$ microfarad, and thus the current will be a quarter of that in parallel, or 0.0117 ampère.

CHAPTER XV.

REGULATING RESISTANCES.

IN order to produce desired variations in the manner of working of various types of electrical machinery, it is sometimes necessary to employ resistance coils connected in circuit with the machine to be regulated. For this and other purposes it may be necessary to provide resistances calculated to effect the following—

- (1) To replace some device cut out of circuit ;
- (2) To regulate and vary the speed of a motor.

(1) The value of a resistance to replace a bank of lamps, an arc lamp, a motor, or other device which requires when working a P.D. of E volts, and takes a current of C ampères, is simply—

$$R = \frac{E}{C} \text{ ohms}$$

This resistance must, of course, be made of such shape, size, and material, that it will be able to carry the current for the length of time it is to be in circuit without overheating. Practice varies considerably in this respect, the rise in temperature allowed ranging from quite a few degrees above the surrounding air to a temperature quite sufficient to discolour paper. No rule can be given for the size of wire or current density, as a great deal depends upon the spacing of the wires, or the degree to which the convolutions of the spirals are separated from one another. With a fair amount of spacing, however, a current ranging from $\frac{1}{8}$ to $\frac{1}{16}$ of that required to melt the wire employed according to the table of fusing currents determined by Mr. Preece, may be taken as reasonable. The materials usually

employed are German silver, platinoid, iron, and manganin, the latter having the advantage of constancy of resistance in spite of variation of temperature.

(2) The resistance employed to vary and regulate the speed of a motor is usually combined with a set of resistances and switch contacts to enable the motor to be gradually introduced into the circuit, so that excessive current through the motor on the one hand, and sudden demand on the generator on the other hand may be avoided.

In all motors the tendency to rotate is due to a pull on the armature conductors by the poles of the magnets, the pull depending directly upon the strength of the current and the strength of the magnetic field. Thus, whatever be the magnitude of the current in the armature, there will be little or no pull on its conductors if the strength of the magnetic field be small. It thus follows that in starting a motor the first thing to see to is production of a strong magnetic field. In the case of a shunt motor, the switch for introducing it into circuit is, therefore, arranged so as to, first of all, make circuit with the shunt coil ; then the armature is introduced into the circuit in series with a considerable resistance, in order that the current may be only a small fraction of that at full load. In this way the two essentials to rotation are produced without any sudden strain either on the motor or generator. In the event of there being no load on the pulley the motor now starts, and will soon reach a speed near the maximum possible ; but if there be a load on the pulley, the torque due to the small armature current may not be sufficient to produce rotation. In this case the switch has to be moved over several contacts, cutting out of circuit a resistance at each step till the current through the armature is sufficient to produce rotation. Regulation of speed is effected in either of two ways, or sometimes by both combined, viz. :—

(a) Reduction of the E.M.F. available for the armature by resistance.

(b) Reduction of the strength of the magnetic field by introduction of resistance into the shunt circuit.

(c) A combination of the two previous methods.

Fig. 39 will illustrate this. The dynamo D has one terminal connected directly to the one brush of the motor M, to which is also connected one end of its shunt coil as usual. The other terminal of the dynamo is connected to a contact arm A, which may be brought into contact with either of the contact blocks 1, 2, 3, 4, 5, 6, or 7, or may be off any of them, when circuit will then be broken. The other brush of the motor is disconnected from the other end of the shunt coil, and is connected to one end of the series of resistance coils placed between the contact blocks; the shunt coil is connected to some part of the resistance coils, as shown at B. If now the contact arm be upon block marked α , there will be no circuit,

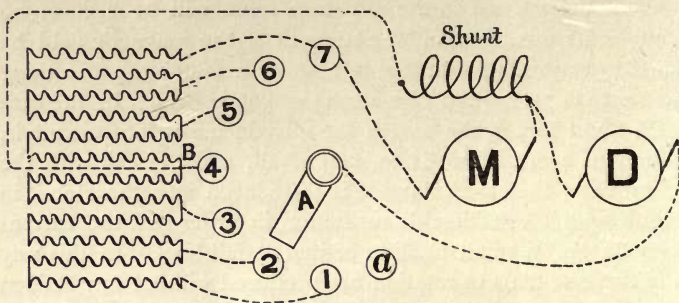


FIG. 39.

but when contact is made with block 1, the armature and shunt coil will then be in circuit, the armature current, however, being quite small as the coils between B and block 1 have a resistance which is large relatively to the armature, but not so large as to make much reduction in the shunt current. Thus there will be produced a considerable field strength, and also some armature current. The resistance now in circuit is such as to prevent the motor running at anything like full speed since the loss of pressure in the resistance coils will very considerably reduce the E.M.F. available for overcoming the full B.E.M.F. of rotation; but on moving the arm A over to block 3 or 4, the resistance in series with the armature is considerably

reduced, and the shunt is now directly connected to the dynamo, this position of the switch being that intended for the slowest speed of running, since the shunt is now strongest, and the armature has still considerable external resistance. It is from this point onwards that the double regulation comes in; for as the arm A is moved onwards towards block 7, the shunt is being made gradually weaker, necessitating an increase of speed in the armature to give the required B.E.M.F., whilst at the same time there is less and less drop of pressure in external resistance till the arm A is on block 7, when the armature then has the full P.D. of the dynamo, and will, consequently, run fastest.

In calculating the magnitude of the resistance coils to be inserted in a switch of the above type it is as well to bear in mind that the current taken by the motor depends mainly upon the demand made upon it at the pulley, and may reach the maximum value even at a slow speed when probably the demand for work may be large. Thus all the coils between blocks 4 and 7 may have to carry the full current, and must consequently be of such a size as to prevent overheating. The coils between contacts 1 and 4 must also be made of such size as will make them quite safe at any current they may have, the largest value of course being such as the full P.D. can produce through the whole external resistance with the armature of the motor stationary; these coils, though not intended generally to be left in circuit for any length of time, are frequently so left, and should therefore not be so slender as to risk fusion. As to the value in ohms, a good deal depends upon the range of regulation required, bearing in mind that in any case such regulation is very wasteful of power since any not used by the motor is dissipated in heating the coils.

Let the value of the resistance between blocks 1 and 4 be called R ohms and that between 4 and 7 = r ohms, the resistance of the armature being R_a and that of the shunt coil R_s . For the present we will leave out of consideration the effect of the resistance in the shunt coil, and deal only with that in the armature.

Calling the dynamo P.D. E volts and the maximum value

of the motor armature current C_a ampères, it is first of all necessary that $R + r$ shall be so large that—

$$C = \frac{E}{R + r} = \text{the starting current}$$

may be less than C_a , or certainly not greater than it. Next we will suppose that the switch has reached the contact block 4, and that the motor is so loaded that it is taking a current of C_a ampères. Under these circumstances there is a loss of pressure in the external resistance and in the armature, neglecting the shunt current, of—

$$C_a(R_a + r) \text{ volts}$$

leaving—

$$E - C_a(R_a + r) = \epsilon$$

the speed being nearly proportional to the B.E.M.F. ϵ . Thus, by making $C_a(R_a + r)$ any desired proportion of E , the speed can be thereby reduced in the same proportion.

Next consider that the arm A has reached the block 7, then the value of ϵ will be simply—

$$E - C_a R_a$$

and the speed will be normal. But with the switch connections as shown in Fig. 39, p. 176, the shunt coil has now a resistance inserted in it, the effect of which is to reduce the magnetizing force, and therefore the magnetic field, but not in the same proportion, since the magnetic circuit is composite, and consequently the various items of it are altering in relative importance. Thus, whereas the air gap was, at full excitation, a certain large proportion of the whole circuit, it is now relatively much more important, since, due to reduction of magnetic density, the permeability of all the iron items has increased. The exact effect of such reduction of magnetizing force is not capable of direct calculation, since the law connecting β and μ is not easily stated in symbols; and, moreover, the greater effect of armature reactions at reduced excitation renders the problem a very difficult one. We can, however, form an approximate estimate of such effect by remembering that, initially, the reduction of lines will be always less than the

reduction of magnetizing force. But, counting armature reactions as more important at low inductions, we shall be very near the truth in assuming that the field falls off directly as the magnetizing force.

Thus in the above case the magnetizing force is proportional to the shunt current, which is as a maximum—

$$C_s = \frac{E}{R_s}$$

and as a minimum—

$$C_{s1} = \frac{E}{R_s + r}$$

If the speed be n at the point when the switch arm is on block 4, when the shunt gets the full P.D., and the armature has the resistance r in series, then this will be the slowest speed if the motor be so loaded as to take the full armature current C_a . But when all the resistance is out of the armature circuit and inserted in the shunt, the speed will have increased from its former value to—

$$n_1 = n \times \frac{E - C_a R_a}{E - C_a (R_a + r)} \times \frac{R_s + r}{R_s}$$

and whence—

$$r = \frac{\left(\frac{n_1}{n} - 1\right)(E R_s - C_a R_a R_s)}{E - C_a R_a + \frac{n_1}{n} C_a R_s}$$

from which it will be seen that in any case the shunt effect is small. But in the simpler case, where the shunt coil is always at the full P.D. of the circuit, then—

$$\frac{n_1}{n} = \frac{E - C_a R_a}{E - C_a (R_a + r)}$$

and—

$$r = \frac{C_a R_a - E + \frac{n_1}{n} E + \frac{n_1}{n} C_a R_a}{\frac{n_1}{n} C_a}$$

In all cases n_1 stands for the maximum speed, and n for the slowest speed. The difference is not considerable between the two cases, the resistance r required to vary the speed from 1 to 2 being about 8 per cent. less when the shunt is connected as shown in Fig. 39, than when the shunt always has the full P.D. at its terminals.

Example LVIII.—A shunt motor having $R_s = 50$ ohms, $E = 100$ volts, $C_a = 100$ ampères, and $R_a = 0.02$ ohm, is required to have a speed ranging from 600 to 1200. Find the value of the regulating resistances required.

(a) When the shunt always has the full P.D. 100 volts;

(b) When the shunt has the P.D. at its terminals varied by the same resistance as in the armature circuit.

Solution.— $\frac{n_1}{n} = 2$, whence in case (a) the resistance to be inserted will be—

$$r = \frac{2 - 100 + 200 + 4}{200} = \frac{106}{200} = 0.53 \text{ ohm}$$

and in case (b)—

$$r = \frac{5000 - 100}{100 - 2 + 10000} = \frac{4900}{10098} = 0.485 \text{ ohm}$$

CHAPTER XVI.

PRIME MOVER COSTS.

IN most forms of heat engine the consumption of fuel may be divided up into two items, one to run the engine idle, and the other to supply the output. The latter is generally represented by a regular constant amount for every actual brake horse-power hour, whilst the former is also generally nearly a constant, bearing some relationship to the difference between full load consumption and idle consumption. The actual consumption for any output is of course the sum of these two. Calling—

B = the maximum brake H.P. of the engine ;

b = consumption per actual B.H.P. hour ;

a = a factor such that $a(bB)$ gives the idle or no load consumption,

we have the actual consumption as being—

$$b(aB + x)$$

where x = the load on the brake in horse-power.

The values of these constants will depend greatly upon the size of the engine, and may be taken to have the following values for steam and gas engines of the usual sizes employed in electric lighting.

FOR STEAM ENGINES.

$a = 0.3$ to 0.4 .

$b = 1.7$ to 3 lbs. of coal per hour.

FOR GAS ENGINES.

$a = 0.16$ to 0.28 .

$b = 12$ to 18 cubic feet of gas per hour,

The smaller the engine in each case the higher the value to be taken.

If, now, we call z pence the price per unit, either pound of coal or cubic foot of gas, of fuel, we can write the cost of one hour as being—

$$zb(aB + x)$$

And the cost of 1 B.T.U. will be found by putting for x the actual B.H.P. required, with the particular gearing and dynamo to give an output of 1 B.T.U.; the cost of 1 B.T.U. thus varying from a somewhat large value at small load to lower values at full load.

Example LIX.—What will be the cost of fuel for 10 B.T.U. from a dynamo of 90 per cent. commercial efficiency, when run at full load of 100^a at 100^b, by a gas engine in which $a = 0.2$ and $b = 15$, the price of gas being 2s. 3d. per 1000 cubic feet? The gas engine has a maximum power of 15 B.H.P.

Solution.—The H.B.P. required will be—

$$\frac{1000}{746} \times 10 \times \frac{100}{90} = 14.74 = x$$

whence the cost of fuel per hour will be—

$$zb(aB + x)$$

where z is—

$$\frac{27}{1000} \text{ penny}$$

or—

$$\begin{aligned} &= \frac{27}{1000} \times 15 (0.2 \times 15 + 14.74) \\ &= \frac{27}{1000} \times 266.1 = 7.185 \text{ pence} \end{aligned}$$

which is the cost of 10 B.T.U., whence the fuel cost per B.T.U. = 0.7185 pence.

Example LX.—If with the above engine the average load is only 3 B.T.U. when the commercial efficiency of the dynamo is only 80 per cent., find the fuel cost of 1 B.T.U.

Solution.—Here the B.H.P. required is—

$$\frac{1000}{746} \times 3 \times \frac{100}{80} = 5.23 = x$$

and the cost of fuel per hour is—

$$\frac{27}{1000} \times 15 (0.2 \times 15 + 5.23) = 3.33 \text{ pence}$$

which is the cost of 3 units or 1.11 pence per B.T.U.

Example LXI.—Find the cost of running the above dynamo by a steam engine having the following particulars. $a = 0.4$ and $b = 2.5$, $B = 15$ when the cost of coal is 0.043 pence per lb.

Solution.—(1) on full load of 10 B.T.U. the B.H.P. as before will be 14.74 whence cost of fuel will be—

$$0.043 \times 2.5 (0.4 \times 15 + 14.74) = 2.23 \text{ pence}$$

or only 0.223 pence per B.T.U.

(2) But on load of 3 B.T.U. the B.H.P. required = 5.23 as before and the cost of 1 hour will be—

$$0.043 \times 2.5 (0.4 \times 15 + 5.23) = 1.207 \text{ pence}$$

or only 0.402 pence per B.T.U.

Example LXII.—A large gas engine having $a = 0.16$, $b = 12$, and of 200 B.H.P., is used to run a dynamo which has a commercial efficiency of 50 per cent. at an output of 7 B.T.U. per hour, and 95 per cent. at 140 B.T.U. per hour. The price of gas is 2s. 6d. per 1000; find the cost of gas per B.T.U. at an output of 7 and 140 B.T.U. per hour.

Solution.—(1) At 7 B.T.U. per hour the horse-power required is—

$$\frac{7 \times 1000}{746} \times \frac{100}{50} = 18.7$$

whence the cost of gas per hour will be—

$$\frac{30}{1000} \times 12 (0.16 \times 200 + 18.7) = 18.28 \text{ pence}$$

or—

$$\frac{18.28}{7} = 2.61 \text{ pence per 1 B.T.U.}$$

(2) At 140 B.T.U. horse-power required is—

$$\frac{140 \times 1000}{746} \times \frac{100}{95} = 198$$

and cost per hour is then—

$$\frac{30}{1000} \times 12(0.16 \times 200 + 198) = 82.8 \text{ pence}$$

or—

$$\frac{82.8}{140} = 0.59 \text{ pence per 1 B.T.U.}$$

It will be seen from the foregoing examples that the cost of fuel will depend very much upon the extent to which the machinery can be loaded. Large engines and dynamos run to supply only a small fraction of their full power will be very wasteful, and so run up the cost of generation. To the cost of the fuel must also be added the cost of attendance, lubrication, etc., all of which items become still more costly as the load goes down. Thus, though at times of full load the cost of fuel may be but a fraction of a penny per B.T.U., the average cost is much increased by the, to a large extent, unavoidable waste at times of light load. The following example will illustrate this.

Example LXIII.—The average pressure generated in a central station is 430 volts, and the system is a five-wire network maintained at 400 volts between the outers. At full load there are 10,000 30-watt lamps in use. The dynamos have an average efficiency of conversion of 92 per cent. The station is on full load for 2 hours, on half load for 8 hours, on $\frac{1}{10}$ load for 6 hours, and runs idle for the rest of the 24 hours. One mechanical brake H.P. costs 1.75 pence per hour for wages, coal, oil, etc., at full load; costs 2.5 pence at half load, and 6 pence at $\frac{1}{10}$ load, whilst at no load the cost of running is

15s. per hour. What should be the price charged per B.T.U., in order that there may be a profit of 10 per cent.?

| Time hrs. | Output watts. | Watts generated. | Brake H.P. required. | Total cost for time in pence. | B.T.U. paid for. |
|-----------|---------------|---|---|-----------------------------------|--------------------------|
| 2 | 300,000 | $\frac{430}{100} \times 300,000$ = 322,500 | $\frac{100}{92} \times \frac{322,500}{746}$ = 469.95 | @ 1.75 pence per hour = 1644.8 | 600 |
| 8 | 150,000 | $\frac{430}{100} \times 150,000$ = 161,250 | $\frac{100}{92} \times \frac{161,250}{746}$ = 234.97 | @ 2.5d. per hr. = 4699.4 | 1200 |
| 6 | 30,000 | 32,250 | 46.995 | @ 6d. per hr. = 1691.82 | 180 |
| 8 | 0 | | | @ 15s. hour = 1440 | 0 |
| | | | | Total cost 9475 pence | Total units sold 1980 |

whence the cost per unit is $\frac{9475}{1980} = 4.78$ pence, and to pay 10 per cent. the price must be 5.258 pence.

EXAMPLES.

CHAPTER I.

1. The suspension of a moving-coil galvanometer is made of platinoid. It is $3\frac{1}{2}$ inches long, 0.0005 inch thick, and 0.004 inch wide. If platinoid will be 20 times the resistance of copper, what is the resistance of this suspension?

2. What relationship will the resistance of the above suspension have to the resistance of the coil, which is 1 inch wide and $2\frac{1}{4}$ inches long, being wound with 400 turns of copper wire 0.002 inch diameter?

3. What will be the E.M.F. required to send a current of 10 amperes through a circuit consisting of a dynamo of resistance of 7 ohms, $1\frac{1}{2}$ mile of leads of No. 6 B.W.G., and 80 lamps each of $2\frac{1}{2}$ ohms resistance? State the loss in the dynamo and in the leads, and give the horse-power of engine required, if the dynamo converts $\frac{9}{10}$ of the power received into electricity. Also if the lamps are paid for at so much per B.T.U. in the lamps, what will be the cost of 1 B.T.U. if a mechanical horse-power costs twopence per hour?

4. What size of leads will be required to keep 400 60-watt 100-volt lamps at a P.D. of 100 volts, if the dynamo is 170 feet from the lamps, has a resistance of 0.019 ohm, and produces a total E.M.F. of 106 volts?

5. How many secondary cells would be required to run the lamps in the above question when each cell has a resistance of 0.00007 ohm, and an E.M.F. of 2.05 volts at the start, but falling to 1.92 volt at the end of discharge? The lamps are connected to the cells by the same leads as above.

6. In the above question the numbers of cells do not come out exactly, but as whole numbers have to be used, find what compensating resistances would be necessary to make the current exactly right.

7. A series dynamo has a resistance of 10 ohms, and sends a current of 12 amperes through 40 arc lamps taking 48 volts each, and equal to 1000 candle-power, all connected in series on a circuit of a total length of $\frac{1}{4}$ mile

of No. 9 B.W.G. What will be the horse-power of engine required if the dynamo has a frictional loss of $\frac{3}{4}$ horse-power?

8. What will be the horse-power of a dynamo to run 40 incandescent lamps in series each of 1000 candle-power, and taking 65 ampères each at 48 volts, if they are on a circuit $\frac{1}{4}$ mile long of No. 3 B.W.G., and the resistance of the dynamo is 1.25 ohm, with a friction loss of $3\frac{1}{2}$ horse-power?

9. A dynamo has an armature resistance of 0.042 ohm, and is required to run 300 16-candle-power 50-watt lamps at the end of leads which are 0.01333 ohm resistance. The total current is 150 ampères. What will be the reading of a voltmeter connected to the terminals of the dynamo, and what percentage is this P.D. of the whole generated E.M.F.?

10. What will be the respective resistances of two bars, one of copper and one of aluminium, but both the same weight and the same length?

11. A pair of conductors, 1 square inch sectional area, conveys 400 ampères to a distance of $\frac{3}{4}$ of a mile from the dynamo. If the load at the far end requires a P.D. 410 volts, find the P.D. required at the dynamo.

12. In a secondary battery the lead lugs connecting cell to cell are $\frac{1}{2} \times 1$ inch in section, and 10 inches long each. If there are 54 cells joined in series, find the resistance of these connections.

13. If the cells in the above question have 15 plates each cell, each plate being 10 inches square and a quarter of an inch distant from the next, find the resistance of the electrolyte in each cell if the resistance of one cubic inch of the solution is 3 ohms.

14. Neglecting the E.M.F. of the cells in the above questions, find the E.M.F. required to send a current of 100 ampères through the mere resistances of the connections and electrolyte if all the cells are in series.

15. The connections on a switch-board are made of rod brass, and have an aggregate length of 27 feet. They are to carry 150 ampères, and the drop of pressure over all is not to exceed half a volt. Find the sectional area of the brass.

16. A standard manganin resistance of 0.001 ohm is 20 inches long. It is made of sheet metal $\frac{1}{16}$ inch thick. How wide will it be?

17. An arc-light circuit consists of 60 lamps in series, each taking 10 ampères. Each lamp has the positive carbon 18 inches long and $\frac{1}{2}$ inch diameter, and the negative carbon 12 inches long and $\frac{1}{16}$ inch diameter. Neglecting the effect of heating, find the loss of energy in the carbons alone.

18. If the arc lamps in the above question be evenly spaced along a circuit at a distance of 30 yards apart, and the cable employed is $\frac{7}{16}$ S.W.G., what will be the total E.M.F. required if each lamp takes 45 volts in addition to the drop in the carbons?

increased temperature due to

19. What will be the nearest cable of strands of S.W.G. copper to make leads to convey 600 ampères a distance of 30 yards so that there shall not be more than 1·26 volt lost in the lead and return?

✓ 20. A motor taking current of 100 ampères at a P.D. of 2000 volts is run from a dynamo 8 miles away. If the loss in the leads is not to be more than 10 per cent., what will be the nearest cable of S.W.G. wire, and what will have to be the P.D. at the dynamo?

✓ 21. If the power in the last question be transmitted at 20,000 volts, what will then be the size of the bare solid conductor employed with only 5 per cent. loss of pressure?

22. Find the horse-power of engine required to run 1000 16-candle-power lamps taking 100 volts and 0·6 ampère each. There is a loss in the leads of 1 volt, and the dynamo gives out $\frac{9}{10}$ of the power received by it.

23. If all the above lamps are replaced by arcs in series, taking 10 ampères and 50 volts each of 900 candle-power, so as to give twice the total light, and if the loss in the leads be 4 volts, and the dynamo gives out 85 per cent. of the power received by it, find the horse-power of engine required, and the number of lamps.

24. In the calibration of an ammeter to read 2000 ampères, what will be the smallest power required, if a number of secondary cells be used in parallel, each cell giving 2 volts?

25. A street 4 miles long is to be supplied with 50 arc lamps per mile, each taking 10 ampères and 45 volts. The current is supplied by 4 dynamos, one to each mile of street, the leads being respectively 1, 2, 3, and 4 miles, lead and return, of $\frac{1}{16}$ square inch sectional area. If the mechanical efficiency of each dynamo be 90 per cent., and its resistance 10 ohms, find the E.M.F. of each, and the power required to drive each.

26. What size of leads will be required to send current to a number of 98-volt lamps from a dynamo giving 200 ampères at 100 volts?

27. A current is required to be sent a distance of 50 yards to 400 lamps, taking 0·6 ampère at 100 volts each, from a dynamo at P.D. of 101 volts. What will be the size of leads required?

✓ 28. How many incandescent, and how many arc lamps, will be required to give together 20,000 candle-power for 20,000 watts? Each arc gives 2000 candle-power for 500 watts, and each incandescent 16 candle-power for 60 watts.

CHAPTER II.

29. The distance between the feeding centres of a two-wire system of distributors is 400 yards, and current is required at the rate of 1 ampère per yard. What will be the size of the distributors if no two lamps are to

differ by more than 2 volts? And what will be the size of the feeders if the loss along them is not to exceed 10 volts, and the station is an average of 500 yards distant from any feeding centre?

30. Two parallel mains running round a district are of $\frac{1}{4}$ square inch sectional area, and are connected to the station by feeders $\frac{1}{2}$ square inch sectional area at points 400 yards apart. Each feeder has to convey 400 ampères to the mains, and there are 5 pairs, of lengths respectively of 100, 200, 300, 400, and 500 yards. If the P.D. at each feeding centre is maintained at 204 volts, what will be the P.D. in the station at the ends of each pair of feeders?

31. In the above circuit, what is the maximum difference in volts between any two lamps at full load?

32. The feeding centres in a two-wire system are 500 yards apart. What must be the sectional area of the distributors, so that there may not be more than 2 volts difference between any two lamps if the full load is at the rate of 1 ampère per yard of street?

33. The distributors in a street 1200 feet long are connected to feeders midway. The load is equal to one 48-watt 100-volt lamp per yard of street, and the feeding centre is maintained at 101 volts. Find the size of the distributors when the furthest lamps are at a P.D. of 99.5 volts at full load.

34. A system of distributors is $\frac{1}{2}$ square inch sectional area, and the load is one 50-watt 100-volt lamp per yard of street. If the feeding centres are maintained at 101 volts, find the distance between them so that the drop in the distributors shall not be more than $1\frac{1}{2}$ volt.

35. A street 500 yards long is supplied with current at the rate of 1 ampère per yard from a 200-volt circuit. What will be the size of leads if the dynamo is connected at the middle of the street, and the difference between any two lamps is not to be more than 3.5 volts? Also, if the loss in the feeders is 7 volts, what will be the horse-power required at the dynamo terminals?

36. In a five-wire system the outers are 1 square inch sectional area. What will be the maximum distance allowable between the feeding centres if no two lamps are to differ by more than $1\frac{1}{2}$ volt? There are an average of four 0.6-ampère lamps in series per yard of street.

37. If the distance between the feeding centres be 400 yards on a three-wire circuit running two 200-volt lamps in series per foot of street, each lamp taking 0.3 ampère, what must be the size of the outers if no two lamps are to differ by more than 2 volts?

38. In a three-wire system running 200-volt lamps, two in series between the outers, the feeding centres are 500 yards apart and supply a maximum current of 500 ampères each. If the outers are $\frac{1}{2}$ square inch sectional area,

what is the P.D. between the terminals of the poorest lamp when the feeding centres are kept at 410 volts?

39. A district is to be supplied with current at the rate of 1 ampère per yard of street, the system is two-wire with 100-volt lamps, and the feeding centres 600 yards apart. What will be the current supplied by each feeder, and what size must the distributors be in order that no two lamps may differ by more than 1 volt?

40. If the feeders in the last question are 700 yards long on an average, and the loss along them is 10 volts, find the weight of copper per mile of street.

41. A district takes $\frac{1}{2}$ ampère per yard of street, using 200-volt lamps on a two-wire system. If the feeding centres are 600 yards apart, what current will they supply, and what will be the size of the distributors in order that no two lamps may differ by more than 2 volts?

42. If the feeders in the above case be 700 yards long, find the weight of copper per mile of street with 20 volts lost in feeders.

43. A district is supplied with two 100-volt lamps in series on a three-wire system at the rate of $\frac{1}{2}$ ampère per yard of street. The middle wire is $\frac{1}{4}$ the section of the outers, and the feeders are 600 yards apart. If no two lamps are to differ more than 1 volt, find the size of the outers. Also find the weight of copper per mile of street, if there be a loss of 20 volts in the feeders, which have an average length of 700 yards.

44. If $\frac{1}{4}$ ampère per yard of street be supplied on a three-wire system with two 200-volt lamps in series, with feeders 600 yards apart, find the size of the distributors so that no two lamps shall differ more than 2 volts. And if 40 volts be lost in the feeders which are an average of 700 yards long, and the middle wire be $\frac{1}{4}$ the area of the outers, find the weight of copper per mile of street.

45. If the system be three-wire, with the outers 1 square inch sectional area, and the middle wire $\frac{1}{2}$ square inch area, the feeders being 600 yards apart, what may be the maximum load in order that no two lamps may differ by more than 2 volts? Each lamp is for 200 volts, and takes 0.3 ampère.

46. A current is required along a street 2000 feet long at the rate of 1 ampère per 10 feet. Find the sectional area of conductors, so that there shall be only 2 volts difference between any two lamps. 1000 feet of copper 1 square inch section is 0.008 ohm—

(a) When current is supplied at one end of the street.

(b) When current is supplied midway. (*City Guilds Examination.*)

CHAPTER III.

47. A secondary battery of 54 cells has a resistance per cell of 0.0002 ohm and an E.M.F. of from 2 volts at start to 1.85 volt at the end of a run. What will be the current sent through a bank of lamps of 2 ohms resistance 70 yards away, connected by leads of 0.02 ohm resistance?

48. Find the speed at start and end of charging for a dynamo to send 150 ampères through the above cells. The dynamo produces $\frac{1}{10}$ volt per revolution per minute, is separately excited, and has armature resistance 0.021 ohm. In charging each cell has B.E.M.F. ranging from 2.2 to 2.4 volts, and the battery is 20 yards away, and connected by leads of 0.0004 ohm per yard.

49. What resistance should be introduced into the circuit on starting to charge in order to keep the current 150 ampères, though the dynamo runs at the higher speed?

50. Find the E.M.F. required to charge 108 cells in two parallel with 200 ampères each cell, if the B.E.M.F. is 2.3 volts each. Each cell has resistance 0.001 ohm, that of the dynamo being 0.005 ohm, and leads 0.01 ohm.

51. Find the E.M.F. required to charge 57 cells in series, each having B.E.M.F. of 2.3 volts and 0.0005 ohm resistance. The dynamo = 0.03 ohm, and leads 0.04 ohm, and current 100 ampères.

52. What will be the electrical efficiency of the above cells when discharging at 100 ampères through lamps on the above leads, each cell giving an average E.M.F. of 1.9 volt? Also state the P.D. at the lamp terminals.

53. If a dynamo gives 100 volts at 1000 revolutions, at what speed must it run to charge 54 cells at 50 ampères, each cell having B.E.M.F. 2.2 volts and 0.005 ohm internal resistance when the leads = 0.01 ohm?

54. If the above cells are discharged at 50 ampères, the E.M.F. of each being 2 volts at start, falling to 1.8 volts at the end of run, what percentage of the power put in is utilized?

CHAPTERS IV. TO IX.

55. What will be the brake horse-power of a motor series wound having armature resistance, 0.025 ohm, series coil 0.025 ohm, when running at 1200 revolutions per minute with a current of 50 ampères supplied at an E.M.F. of 100 volts? The electrical loss in the machine is $\frac{1}{3}$ of the friction losses.

56. In the above motor what will be the value of the total flux through

the armature if there be 360 wires all round? Also find the density in the air-gap, if the polar angle is 135 degrees, the bore is 22 cm. diameter, and length 20 cm.

57. In the above motor, what will be the pull in pounds on any one wire?

58. A motor is supplied with 150 ampères at 100 volts. The magnets are shunt wound to 40 ohms, and the armature resistance 0.01 ohm. If 1 horse-power is lost in various frictions, what will be the brake horse-power, and what the mechanical efficiency?

59. If the above motor armature has 120 wires all round, find the total lines, and the density in the air-gap, if the polar angle is 140 degrees, the diameter and length of bore respectively 30 cm. and 25 cm., and the speed 1200 per minute.

60. A machine separately excited gives 100 volts P.D. at 15 revolutions per second when run as a dynamo. The armature resistance is 0.015 ohm, the watts lost in eddy currents = 120, in hysteresis = 180, and in mechanical friction = 150. At what E.M.F. and current will it run, at the same speed as an unloaded motor?

61. A magneto machine gives 50 volts as a dynamo at no load. The armature resistance is 1 ohm, and 125 watts are lost in various frictions. Find the E.M.F. and current required to run it as an unloaded motor at the same speed.

62. A shunt dynamo having armature resistance 0.02 ohm, shunt resistance 40 ohms, when run on open circuit at 20 revolutions per second, gives P.D. of 100 volts. If it be separately excited at 100 volts, and run as a motor at the same speed, to work a crane lifting 3 tons, at the rate of 30 feet per minute, what will be the current taken, and E.M.F. required, if the crane has a friction loss at the rate of 200 pounds per ton lifted, and the mechanical friction of the motor is equal to 150 watts, the hysteresis loss 130 watts, and eddy current loss 170 watts? Assume that there is no armature reaction.

63. What is the commercial efficiency of the above combined crane and motor?

64. The E.M.F. produced by a separately excited machine when run as a dynamo is proportional to the speed, and is 100 volts at 1000 revolutions per minute. The armature resistance is 0.04 ohm. The loss due to hysteresis is proportional to the speed, and is 120 watts at 1000 per minute. The eddy current loss, however, is 80 watts at 1000 revolutions per minute, and is proportional to the square of the speed. The mechanical-friction loss varies directly as the speed, and is 110 watts at 1000 revolutions. If the above machine be supplied with current at 100 volts, at what speed will it run, and what current will it take (a) unloaded, (b) loaded to 2 brake horse-power?

65. A machine has armature resistance of 0.005 ohm, and shunt resistance 25 ohms, and is separately excited at 100 volts. If run at 600 per minute as a dynamo, it gives 100 volts on open circuit, the eddy current loss being 250 watts, the mechanical friction 300 watts, and hysteresis loss 400 watts. When supplied at 100 volts, what will be the speed and current required to run as an unloaded motor?

66. A series dynamo gives a current of 12 ampères, and is required to run a series motor of 1 brake horse-power. The dynamo has 360 wires all round, and a resistance of 1.3 ohm, with one million lines through the armature. The resistance of the motor is also 1.3 ohm, and $\frac{1}{2}$ horse-power is lost in various internal frictions. At what speed must the dynamo run, and what will be the P.D. at the motor terminals, the electrical efficiency of the dynamo, and the electrical and mechanical efficiency of the motor?

67. A series motor produces a B.E.M.F. of 94 volts, and is supplied with 100 ampères at a P.D. of 100 volts. If $\frac{1}{2}$ horse-power is lost in various frictions, what will be the brake horse-power, and the electrical and mechanical efficiency?

68. A motor has an armature resistance of 3 ohms, with 3.6 million lines through it. The shunt coil is 750 ohms, and $\frac{1}{2}$ a horse-power is lost in various frictions. If the armature has 720 wires all round, and the machine is supplied with 15 ampères at 550 volts, find the speed, the brake horse-power, and the electrical and mechanical efficiency?

69. A series motor has 240 wires all round the armature, with a resistance of 0.08 ohm, and 22 million lines through it. The magnet winding is 0.12 ohm, and the machine is to give 100 brake horse-power when taking a current of 150 ampères. If n be the revolutions per second, and the mechanical friction loss is $100n$, the hysteresis loss $80n$, and the eddy current loss is $3n^2$, find the E.M.F., the speed, and electrical and mechanical efficiency.

70. A series machine has 140 wires all round the armature, which is 222.2 square centimetres sectional area, with a density of $18,000$, and a speed of 1200 per minute. If the P.D. at the terminals for a current of 100 ampères is 106 volts, what would be the electrical efficiency as a dynamo, and also as a motor at the same speed and current? Also, if the friction loss is $\frac{2}{3}$ horse-power, in both cases, state the brake horse-power as a motor and mechanical efficiency, both as dynamo and motor.

71. A series dynamo, when giving a current of 10 ampères, has a P.D. at its terminals of 1000 volts at a speed of 1040 per minute. The armature resistance is 1.8 ohm and magnet resistance 2.2 ohms. At what speed will it run as a motor if supplied with 10 ampères at a P.D. of 1000 volts? The internal friction losses are $\frac{1}{2}$ horse-power. Also give electrical efficiency as a dynamo, the brake horse-power and electrical and mechanical efficiency as a motor.



72. What would be the E.M.F. required to run the above motor at 1040 revolutions per minute? and what would then be its brake horse-power and electrical and mechanical efficiency, the internal friction being 400 watts?

73. An armature running at 20 revolutions per second has 96 wires all round, and 4 million lines through it; its resistance is 1.05 ohm, and the shunt magnet coil 20 ohms. If 200 ampères be passed through the armature, what E.M.F. will be required to run it as a motor? Also if the hysteresis and eddy current losses are together equal to 300 watts, and $\frac{1}{2}$ a horse-power is lost in mechanical friction, what will be the brake horse-power, and the electrical and mechanical efficiency?

74. A crane is to lift 1 ton at the rate of 500 feet per minute, and has a friction loss of 5 horse-power. It is run by a motor having 6 million lines through the armature, which has 400 wires all round, and a resistance of 0.05 ohm. There is a loss of $\frac{2}{3}$ of a horse-power in mechanical friction, and 500 watts in iron losses. The magnets are series wound to 0.08 ohm. If it is supplied at 600 volts, what will be the current taken, the speed, the brake horse-power, the electrical and mechanical efficiency of the motor, and the plant efficiency?

75. A motor with ring armature $16\frac{1}{2}$ inches mean diameter and 120 wires all round runs in a field of 3000 lines per square centimetre at 800 revolutions per minute. The length of the bore is $16\frac{1}{8}$ inches, its diameter being 17 inches, and the curved length of the polar face 22 inches. The resistance of the armature is 0.024 ohm, the shunt coil being 48 ohms. Half a horse-power is lost in mechanical friction, and 400 watts in iron losses. If the armature be supplied with a current of 200 ampères, what will be the pull on one side of the belt on a pulley 1 foot mean diameter? Also give the brake horse-power and the electrical and mechanical efficiency.

76. A mechanical contrivance requires 100 horse-power, and wastes 15 horse-power in friction. If run by a series motor having armature resistance 0.06 ohm, magnet coil 0.08 ohm, 360 wires all round the armature and 18 million lines through it, $1\frac{1}{4}$ horse-power lost in friction, 600 watts in iron, and supplied with a P.D. of 700 volts, find the speed, the current taken, the electrical and mechanical efficiency of the motor, and the plant efficiency.

77. A compound dynamo to give P.D. of 105 volts and a current of 90 ampères is to have 92 per cent. electrical efficiency. If N be about 3 million lines, the speed 1100 per minute, and half the loss is to be in the armature, the rest in the magnets divided as $\frac{2}{3}$ in the shunt and $\frac{1}{3}$ in the series coil, find the wires all round, and the resistances of the armature, shunt, and series coils.

78. A shunt motor has the shunt resistance 750 ohms, armature resistance 2 ohms, and N 4.2 million. If supplied with 15 ampères at 500

volts, find the brake horse-power, the speed, and the electrical and mechanical efficiency, if there be 720 wires all round the armature, and the friction losses are equivalent to 300 watts.

79. The magnetic circuit of a dynamo has reluctance = 0.0025 , and an average of 5 million C.G.S. lines through it. Find the size of wire for a shunt winding, if the average length of one turn is 30 inches, and the P.D. at the terminals is 400 volts.

80. In the above question, what number of turns will be required on the shunt coil so that the energy waste shall not exceed 600 watts?

81. A dynamo takes 8000 ampère turns, and produces a terminal P.D. of 100 volts. If the average length of one turn on the shunt coil is 2 feet, find the size of wire required.

82. An armature running at 18 revolutions per second, has 288 wires all round, and a resistance of 0.025 ohm. Find N so as to give 70 ampères at 100 volts.

83. The electrical efficiency of a dynamo is 98 per cent., and mechanical efficiency 93 per cent. If the output be 200 units, what power is wasted in resistance, and what power in mechanical and other frictions?

84. A drum armature is 3 feet in diameter, with a 10-inch hole and 3.8 feet length of iron parallel to the shaft. It runs at 400 revolutions per minute, and is worked at such a density that the hysteresis loss is 5000 ergs per cubic centimetre per cycle. Find the hysteresis loss in watts.

85. If the resistance of the above armature be 0.005 ohm, the current 600 ampères at P.D. of 440 volts, shunt resistance 110 ohms, eddy current loss 2 horse-power, and mechanical friction $2\frac{1}{2}$ horse-power, find the mechanical efficiency.

86. A shunt dynamo has electrical efficiency 95 per cent., and gives an output of 20 units. If there is an equal loss in the shunt and in the armature, and the P.D. is 100 volts, find the resistances of the armature and shunt coils.

87. If the mechanical efficiency of the above dynamo is $\frac{9}{10}$, what power must be wasted in other ways than in resistance?

88. A shunt dynamo has magnet coil 33 ohms resistance, and armature 0.005 ohm, and gives current to 500 60-watt 100-volt lamps, with a terminal P.D. of 101 volts. Find the electrical efficiency; and if the mechanical efficiency is 81 per cent., what horse-power of engine will be required, and what power is lost otherwise than electrically?

89. Two series machines, each having armature resistance 0.025 ohm, and magnet resistance 0.025 , also N is 5 million lines, and the current 100 ampères. They are connected together by leads of 0.1 ohm resistance. Each machine has a frictional loss of 600 watts, which may be considered to be independent of the speed. If one machine be run as a

dynamo by an engine of 15 brake horse-power, at what speed will they run, and what will be the brake horse-power of the motor when the current is 100 ampères? Also state the ratio of the power given out by the motor to the power received by the dynamo.

90. In the last question, what brake horse-power engine would be required to run the dynamo if the motor be used so as to give out 15 brake horse-power, the current and losses being the same as before? Also state the speeds.

91. A series motor is to give 50 brake horse-power at a speed of 600 revolutions per minute. Its armature has 720 wires all round, is $\frac{2}{3}$ ohm resistance, and has 12 million lines through it. If the field coils have a resistance of 0.833 ohm, find the E.M.F. required to supply the above power through leads $2\frac{1}{2}$ ohms resistance, the friction losses being 2 horse-power.

92. How many ampère-turns will be required on each limb on a Manchester-type machine having $N = 4.2 \times 10^6$, $A_a = 280$, $l_a 35$, $A_g 1620$, $l_g 2.1$, $A_p 587$, $l_p 35$, $A_m 294$, $l_m 26$, $\nu = 1.5$? The pole-pieces are of cast iron, and magnet cores wrought iron.

93. A shunt dynamo to give 310 volts has the length of the average turn on the magnet coils 1 metre, and is of the single-magnet type. It also has the following dimensions: $l_a 70$, $A_a 1400$, $N 21.6$ millions, $l_g 2.4$, $A_g 4000$, $l_m 240$, $A_m 2700$. If $\nu = 1.4$, and the magnets are wrought iron wound with 2000 turns of wire, what will be the shunt resistance at a temperature of 35°C .?

94. A single-magnet bipolar dynamo has the pole-pieces of cast iron and yoke of wrought iron, on which is placed the magnet winding. The armature resistance is 0.024 ohm hot, with 144 wires all round, and $N = 7$ million lines at no load. It is to give 200 ampères at a P.D. of 120 volts, and has the following dimensions: $l_a 40$, $A_a 450$, $l_g 2.5$, $A_g 2300$, $l_p 80$, $A_p 1300$, $l_m 76$, $A_m 700$, with $\nu = 1.43$. Its shunt coil has 4000 turns, each of 45 inches average length, and the series coil is to be 0.01 ohm resistance hot. If the brushes are lead forward 6 wires on the armature at full load, find the windings of the series coil, the size of the shunt wire, and the electrical efficiency at full load. The shunt coil is connected "long shunt."

95. An armature, gramme wound, for a bipolar-field is required to carry 100 ampères at a P.D. of 100 volts. It has the following dimensions: $N = 4$ millions, $n 20.43$, $w 126$ each, being 80 cms. average length and 0.24 sq. cms. sectional area. The iron stampings are 25 cms. diameter with 15-cm. hole, and make altogether 25 cms. length of iron parallel to the spindle. If the over-all surface is 3675 sq. cms., what will be the rise in temperature above the air on account of copper and hysteresis losses, neglecting the heating due to eddy currents? Take 1 cc. of copper to be 0.000002 ohm resistance.

96. A drum armature for bipolar-field is 25 cms. diameter with 7-cm. hole, and has 25 cms. length of iron parallel to the shaft. It runs at 20'43 revolutions per second in a field of 4.95×10^6 lines, and is wound with 102 wires all round, each of 0.3 sq. cm. sectional area. Each convolution has an average length of 135 cms. If $\rho = 0.000002$, and there be the same rise in temperature as in the gramme armature above, find the maximum current allowable, neglecting the eddy current loss.

97. The magnetic system of the gramme armature in question 95 above has the following dimensions: l_a 31, A_a 250, l_g 2, A_g 950, l_m 170, A_m 415, all the iron parts being charcoal iron. The leakage factor is 1.4. Find the size of shunt wire required if the average turn is 3 feet long, and the P.D. at the terminals is 100 volts, the maximum temperature of the shunt coil being 35° C.

98. Find the size of shunt wire for a dynamo having the following dimensions: Armature ring wound, $w = 252$, l_a 37, A_a 137, diameter of core $10\frac{1}{2}$ ins., bore $11\frac{1}{2}$ ins., and length of 9 ins. Magnet limbs are of cast iron, 9 ins. \times 7 ins. area and $6\frac{1}{2}$ ins. apart, with no polar projections, l_m 100. Speed 1000 per minute to produce 97 volts on open circuit. One turn on the shunt coil is 40 ins. long, and the coils rise to a temperature of 40° C. Leakage factor $1\frac{1}{2}$.

99. If the armature resistance in the above machine is 0.07 ohm, and the shunt coil 42 ohms, find the number of series turns to keep the P.D. of 97 volts at 70 ampères. Allow the series coil to be 0.04 ohm resistance, and the lead of the brushes 20 degrees. The connections are to be "long shunt."

100. A dynamo is to generate 118 volts, and has 17,080 lines per square centimetre in the armature, which has an area of 480 sq. cms., and runs at 720 revolutions. The average length of 1 coil on the (ring wound) armature is 46 ins., and the size of conductor 0.072 sq. in. If the machine be bipolar, and gives a current of 200 ampères, find the P.D. at the terminals at 35° C.

101. A dynamo has 200 wires all round the armature, which has a resistance of 0.03 ohm, an area of 250 sq. cms. at a density of 17,000, and is to run 200 half-ampère 16-candle-power lamps at the end of leads of $\frac{1}{2}$ sq. in. area and 20 yards long. At what speed must she run to keep the P.D. at the lamps at 100 volts?

102. A shunt dynamo gives a terminal P.D. of 216 volts at a current of 50 ampères. Its armature has 7 million lines through it, and 160 wires all round, each coil being 4 feet long, and bipolar ring wound. It runs at 1200 revolutions, and the shunt resistance is 108 ohms. Find the size of wire on the armature at a temperature of 35° C.

CHAPTER X.

103. A voltmeter coil has a core $\frac{1}{2}$ inch diameter, and can be wound to a depth of $\frac{1}{4}$ inch. It is 1.2 inch long, and requires about 300 ampère-turns to produce a deflection to the full extent of the scale. What size of wire will be required to make the instrument read 120 volts total, and what will be the energy wasted in the coil if wound with copper wire covered with silk to a radial depth of 0.001 inch?

104. A certain type of instrument wound as an ammeter to read 20 ampères total has 40 turns of wire on its coil. The length of the mean turn is 6 inches, and the diameter of the wire 0.2 inch. What is the waste of energy at full reading?

105. If the space for winding in the above instrument is 2 inches by $1\frac{1}{4}$ inch, find the size of wire and total waste of energy to make it read as a voltmeter to 20 volts with the coil wound with copper, and having an extra coil to increase the reading up to 120 volts wound with manganin.

106. How much will it cost a central station to keep an indication of 105 volts on each of two voltmeters for a year of 8000 hours, if the cost of a B.T.U. is $2\frac{3}{4}$ pence, and interest on the first cost of instrument is to be at the rate of 5 per cent. per annum?—

(1) A magnetic instrument costing £10, and having a resistance of 2100 ohms.

(2) A hot-wire instrument costing £7, and having a resistance when in use of 470 ohms.

107. An instrument is wound with copper wire, and has a resistance of 1855.7 ohms at 0° C. Its dial is divided into 120 divisions, and 37.5 milli-ampères gives a deflection of 75 divisions. What will be the constant of this instrument as a voltmeter at 20° C., and what resistance should the extra coil of manganin have to increase the range to 2400 volts? Also state the T.V.R. with the extra coil in circuit if manganin is of constant resistance.

108. A voltmeter is wound with copper, and has a resistance of 2000 ohms at the temperature at which it was calibrated to read 120 volts total. It is first connected to the terminals of a dynamo in a hot engine-room, where its resistance rises to 2200 ohms. It is then taken outside, where the temperature is so low as to make its resistance only 1800 ohms, and connected to the end of the circuit, but on the dynamo side of the load. The leads have a resistance of 0.02 ohm, and the lamps take 100 ampères. If the true P.D. at the dynamo is 102 volts, what will the instrument read in the two cases?

109. A voltmeter of 1500 ohms resistance is connected to the terminals

of a lamp. An ammeter connected in the main circuit reads 0.6 ampère. If the voltmeter indicates 150 volts, what is the resistance of the lamp?

110. An instrument reads 10 milli-ampères for 100 divisions deflection at 15° C., being wound with copper to a resistance of 100 ohms. What will it read as a voltmeter at 25° C., when in circuit with a manganin extra coil of 2900 ohms resistance?

111. A voltmeter to read 100 volts requires 1000 ampère-turns. If the average turn of wire be 3 inches long, find the size of wire required.

112. Find the diameter of copper wire for a voltmeter requiring from 300 to 400 ampère-turns, reading 60 volts total, if the mean diameter of the winding is $\frac{3}{4}$ inch.

113. A voltmeter requires 750 ampère-turns, and reads 120 volts total. If the mean turn is 6 inches long, find the diameter of copper wire required.

114. If only one-fourth part of the resistance of the above instrument is to be upon the working coil, the rest being of manganin on an extra coil, find the diameter of the copper wire required for the instrument and the total resistance, if the waste is 8 watts.

CHAPTER XI.

115. A train weighs 120 tons, and is to go at the rate of 25 miles per hour. The resistance to motion on the level is 25 lbs. per ton; the gearing is such that 1 ampère gives a pull of 10 lbs. The line is divided into four stages, viz. level, up an incline of 1 in 480, up incline of 1 in 348, and down incline of 1 in 960. Find the current required for each stage, going and returning, and also state the horse-power.

116. The resistance to motion on the level is 30 lbs. per ton, and the train weighs 100 tons. 1 ampère gives a tractive force of 20 lbs. Find the current, the E.M.F., and horse-power required to run both ways on the line in the previous question.

117. If the E.M.F. of supply is 500 volts, find the current required to move a train of 200 tons weight at a speed of 50 miles per hour both on the level and up and down an incline of 1 in 200. The resistance to motion on the level is 15 lbs. per ton.

118. In the last question find the total current to get up full speed in five minutes.

119. Find the current required to run a car of 12 tons weight at 10 miles per hour on the level and up and down inclines of 1 in 200 and 1 in 80, if the resistance to motion on the level is 25 lbs. per ton, and the E.M.F. of supply is 100 volts,

120. For the above car find the extra current required to get up full speed in half a minute.

121. A car weighing 10 tons runs up an incline of 1 in 100. The motor is geared to give a pull of 10 lbs. per ampère. If the resistance to motion on the level be 30 lbs. per ton, find the current required. Also give the horse-power for a speed of 4 miles per hour.

122. A tram-car, weighing 6 tons empty, starts empty, on a level line, picking up 20 persons halfway along. On reaching an incline up 1 in 50, 10 more persons get on. Fifteen persons get off when the car is going down 1 in 100. What will be the current required on the level both empty and loaded; on incline up 1 in 50 loaded; and down 1 in 100 loaded as above? The tractive force required is 25 lbs. per ton on the level, and 1 ampère gives a pull of 8 lbs. Also state the horse-power required for the various conditions for a uniform speed of 6 miles per hour. Take 20 persons to one ton.

123. A series dynamo has the following relationship between N in the armature and the current C :—

| C | 2 | 4 | 6 | 8 | 10 | 12 | 13 | 14 | 16 | 18 |
|---|------|-----|------|-----|-----|------|------|------|------|-----|
| N | 0.90 | 1.5 | 1.85 | 2.1 | 2.3 | 2.44 | 2.45 | 2.44 | 2.26 | 1.8 |

where N is in millions of lines. The armature has 500 wires all round, and a resistance of 0.77 ohm. The pull exerted as a motor is $4 \times C \times N \times 10^{-6}$ lbs. If supplied with an E.M.F. of 300 volts, find the current required as a motor to haul a car 3 tons in weight on level, and up and down incline of 1 in 150. Also give the speed of the car and the horse-power required. The resistance to traction on the level is 25 lbs. per ton.

124. A series dynamo of 0.15 ohm resistance is supplied with current at 440 volts, and has the following relationship between C and N :—

| C | 10 | 26 | 50 | 75 | 100 | 120 | 140 | 150 | 160 | 180 | 200 | 220 | 240 | 250 |
|---|----|----|----|------|------|------|------|------|------|------|------|------|------|-----|
| N | 4 | 8 | 12 | 14.8 | 16.7 | 17.7 | 18.1 | 18.2 | 18.1 | 17.5 | 16.6 | 15.3 | 13.4 | 12 |

the induction being in millions of lines. If the tractive force exerted by it is $0.5CN \times 10^{-6}$ lbs., find the C required to run a train of 50 tons weight on level line, taking 20 lbs. per ton; the maximum speed in miles per hour, and the revolutions per second of the motor.

125. With the above motor and train, give the C required, the maximum speed of train and of motor to go up an incline of 1 in 200.

126. For the same motor and train, find the maximum speed of train, the C required, and the speed of the motor to go down an incline of 1 in 200.

127. Again, with the same train, find the shortest time in which the maximum speed can be attained on the level line.

128. A shunt motor, working at 100 volts, has 200 wires all round the armature and the following relationship between the current in the armature C_a and the magnetic flux N :—

| C_a | 6 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 86 |
|-------|-------|--------|-------|------|--------|--------|--------|--------|------|--------|
| N | 2'485 | 2'4825 | 2'475 | 2'46 | 2'4375 | 2'4023 | 2'3525 | 2'2823 | 2'17 | 2'0075 |

N being given in millions of lines. The armature resistance is 0'12 ohm, and that of the shunt 50 ohms. The pull exerted is $0'8CN \times 10^{-6}$ lbs. Find the motor speed, the current required, and the speed of the car to run on the level if the car weighs 4 tons and resistance to motion is 25 lbs. per ton.

129. With the same motor and car, find the current required, speed of car and motor to go up incline of 1 in 200.

130. For what current will the speed of the above motor be slowest?

CHAPTERS XII. TO XIV.

131. The maximum value of the alternating E.M.F. in a circuit is 2820 volts, and the resistance is 100 ohms, being non-inductive. What will be the average value of the current; and if the terminals of the circuit be connected to an electrostatic voltmeter, what will it indicate?

132. A transformer has a primary resistance 12 ohms, a secondary resistance 0'02 ohm. It is supplied with a virtual P.D. of 2010 volts at the primary. What will be the P.D. at the secondary terminals when running 50 lamps taking 0'6 ampère each, the ratio of transformation being 20 to 1, and the magnetizing current negligible?

133. A circuit has 10 ohms resistance and a B.E.M.F. of 20 volts: what will be the current through it if connected to an alternating P.D. of 29 volts as indicated on a hot-wire voltmeter?

134. What will be the horse-power supplied to a circuit having a B.E.M.F. of 100 volts virtual, and a resistance of 1 ohm, when connected to a dynamo producing a maximum E.M.F. of 200 volts alternating?

135. A circuit has a resistance of 0'015 ohm, and takes a current of 50 ampères when connected to an alternating P.D. of 60 volts. Find the B.E.M.F.

✓ 136. An alternating P.D. of 100 volts virtual is supplying a current of 12 ampères to two 40-volt lamps connected by a switch with two choking coils, in such a manner that when the switch is on contact No. 1, the two lamps in series are in series with coil x ; but when the switch is on contact

No. 2, only one lamp is in circuit, and is in series, with the two coils x and y in series. Find the B.E.M.F. which the coils x and y must have.

137. What must be the maximum magnetic flux in the cores of the coils x and y in the previous question, if they have each 200 convolutions of wire, and negligible resistance, there being 80 complete cycles per second?

138. A transformer is to supply current to one hundred 60-watt 100-volt lamps, and the secondary resistance is 0.107 ohm. The primary has 20 times as many convolutions as the secondary, and its resistance is 6 ohms. The magnetizing current is 0.8 ampère, and lags practically 90° after the primary E.M.F. What must be the P.D. at the primary terminals to give the lamps 100 volts?

139. A transformer is working off mains kept at a P.D. of 1015 volts. The resistance of its primary coil is 13 ohms, and that of the secondary 0.11 ohm. If the ratio of transformation is 10 to 1, what will be the P.D. at the secondary terminals, when the secondary current is 20 ampères, and the primary magnetizing current is 0.5 ampère lagging 90° ?

140. With the above transformer, what should the primary terminal P.D. be to make the secondary P.D. exactly 100 volts?

141. A transformer with 9.6 to 1 ratio of transformation is to give 100 ampères at a secondary P.D. of 205 volts. The secondary resistance is 0.035 ohm, and that of the primary 3 ohms. It is connected to the station by leads having a total resistance of 4 ohms. Find the P.D. at the station end, at zero load, half-load, and full load, when the magnetizing current may be neglected.

142. A transformer has an output of 10 units at full load, with a loss of 0.1 unit in the iron, and 0.1 unit in the copper. What will be the all-day efficiency if run on a load factor of—

| | | | | | | |
|-------|----|---------------|---------------|---------------|---------------|---|
| Hours | 10 | 4 | 6 | 2 | 1 | 1 |
| Load | 0 | $\frac{1}{4}$ | $\frac{1}{3}$ | $\frac{1}{2}$ | $\frac{3}{4}$ | 1 |

143. A transformer has to convert from 1000 volts to 200 volts, giving a secondary current of 60 ampères. The primary resistance is 0.75 ohm, the secondary being 0.021 ohm. What will be the efficiency at $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, and full load, if there be an iron loss of 120 watts?

144. If the above transformer be used on a load factor thus—

| | | | | | | |
|-------|----|---------------|---------------|----------------|---------------|---------------|
| Hours | 16 | $\frac{1}{2}$ | $\frac{1}{2}$ | 2 | 3 | 2 |
| Load | 0 | 1 | $\frac{3}{4}$ | $\frac{6}{10}$ | $\frac{1}{2}$ | $\frac{1}{4}$ |

what will be its all-day efficiency?

145. The primary coil of a closed iron circuit transformer is to work at P.D. of 2050 volts. The secondary is to give a maximum of 50 ampères, at a P.D. of 101 volts. The secondary resistance is 0.018 ohm, and the primary 8 ohms. If the iron loss is 140 watts, what will be the efficiency of conversion at full load, $\frac{1}{2}$, and $\frac{1}{10}$ load? And if employed on a load diagram as under, what will be the all-day efficiency?—

| Hours | $\frac{1}{2}$ | $\frac{1}{2}$ | 1 | 2 | 4 | 2 | 14 |
|-------|---------------|---------------|------|-----|-----|-----|----|
| Load | 1 | 0.9 | 0.75 | 0.5 | 0.4 | 0.2 | 0 |

146. A transformer has a secondary current of 400 ampères, the resistance of the secondary 0.0015 ohm, and the primary 0.8 ohm, with 20 times as many turns as the secondary. If the iron loss is two-thirds of a horse-power, and the magnetizing current is 5 ampères lagging nearly 90° after the primary E.M.F., what will be the efficiency at full, $\frac{1}{2}$, $\frac{1}{10}$, and $\frac{1}{20}$ load?

147. A number of open iron circuit transformers are supplied by a concentric cable 5 miles long, and having a capacity of 2 microfarads per mile. The total magnetizing current is 20 ampères lagging practically 90° . The cable has a resistance of 4 ohms, and the P.D. at the transformers is to be 2020 volts. Find the P.D. in the station and the capacity of the shunting condenser required to reduce the current in the cable to practically zero. The dynamo is 140 ϕ per second.

CHAPTER XV.

148. A series motor which requires 50 ampères at 400 volts has a resistance of 0.8 ohm. It is to have a starting and regulating resistance such that the starting current cannot exceed 30 ampères, and the speed may range from 200 to 800. Find the values of the resistances required, and state the maximum power for each speed with a current of 50 ampères.

149. A shunt motor having R_s 35, R_a 0.05, and running at 800 when supplied at 120 volts, is required to range in speed from 100 to 800. The full-load current is 100 ampères. What will be the value of the regulating resistance required, arranged so that the shunt is always at a P.D. of 120 volts? Also, if there be a friction loss of 800 watts proportional to the speed, what will be the brake horse-power at slow and high speeds with full current in each case?

150. A shunt motor is to have a regulating resistance arranged so as to insert resistance in the shunt circuit as it is removed from the armature circuit. R_s = 25, R_a = 0.02, and the full-load current is 250 ampères at 100 volts. The speed is to range from 200 to 600. Find the value of the resistance.

CHAPTER XVI.

151. A gas-engine consumes when on full load three times as much gas as when running idle. At full load it gives 5 horse-power at the pulley, and runs a dynamo having R_s 50 ohms and R_a 0.3 ohm, giving 100 volts. Find the average cost of 1 B.T.U. for a run of 12 hours, if on no load 2 hours, $\frac{1}{4}$ load 3 hours, $\frac{1}{2}$ load 6 hours, and full load of 40 sixty-watt lamps for 1 hour. The dynamo has a friction loss of 300 watts, and each brake horse-power on the engine takes 18 cubic feet of gas costing 3 shillings per 1000.

152. A steam-engine has a no-load loss equivalent to $\frac{3}{10}$ its maximum brake power, which is 200 horse, for each of which the consumption of coal is 1.8 lb. per hour, costing 15 shillings per ton. The friction load attached to it is 20 horse-power, and useful load 25 ampères at 400 volts for 5 hours. Find the average cost of 1 B.T.U. during the run.

ANSWERS.

1. 23·45 ohms.
2. $\frac{1}{23}$ R of coil.
3. 2089·6 volts, 7 volts in dynamo, 19·6 volts in leads, 31·12 H.P.,
3·112 pence.
4. 0·4557 sq. in.
5. 53 or 54.
6. 0·000916 ohm at start, and 0·00555 ohm at finish.
7. 33·69 H.P.
8. 179 H.P.
9. 102 volts ; 94 per cent.
10. 2 to 1.
11. 435·472 volts.
12. 0·018 ohm.
13. 0·000535 ohm.
14. 4·7 volts.
15. 0·272 sq. in.
16. 5·76 ins.
17. 1746 to 3492 watts.
18. 2894 or 3070 volts.
19. $\frac{61}{11}$ S.W.G.
20. $\frac{91}{13}$ S.W.G. ; 2185·6 volts.
21. About No. 13 S.W.G.
22. 90·26 H.P.
23. 28·5 H.P., 36 lamps.
24. 5·3 H.P.
25. E.M.F. 2364, 2378, 2392, and 2406 volts ; power 35·2, 35·4, 35·6,
35·8 H.P.
26. 0·804 sq. in.
27. 0·578 sq. in.
28. 8 arcs, 272 incandescents.
29. 0·4824 sq. in., 0·9648 sq. in.
30. 207·86, 211·72, 215·58, 219·44, 223·29 volts.
31. 3·86 volts.
32. 0·7545 sq. in.

33. 0'3087 sq. in.
34. 498 yds.
35. 0'432 sq. in., 138'7 H.P.
36. 1290 yds.
37. 0'22 sq. in.
38. 202'73 volts.
39. 600 amps., 2'17 sq. ins.
40. 83'3 tons.
41. 300 amps., 0'5427 sq. in.
42. 20'7 tons.
43. 0'5427 sq. in., 21'93 tons.
44. 0'144 sq. in., 5'69 tons.
45. 1'82 amps. per yard of street.
46. 1'6 sq. in., 0'4 sq. in.
47. 53'1 amps. to 49'1 amps. at end of run.
48. 1259'7 and 1367'7 per min. at end.
49. 0'072 ohm.
50. 141 volts.
51. 140'95 volts.
52. 93'6 per cent., 101'45 volts.
53. 1328 per min.
54. 77'2 per cent.
55. 6'03 H.P.
56. 2291 per sq. cm.
57. 0'2566 lbs.
58. 18'47 H.P., 91'8 per cent.
59. 4,105,208; 4032.
60. 100'0675 volts; 4'5 amps.
61. 56'25 volts and 2'5 amps.
62. 101'08 volts, 56'62 amps.
63. 79 per cent.
64. 998'76 per min. and 3'1 amps.; 992'752 per min. and 18'119 amps.
65. 599'715 and 9'5 amps.
66. 30'05 per sec.; 90'18 volts, 83'3 per cent., 82'7 per cent., and 68'9 per cent.
67. 12'1 H.P., 94 per cent., and 90'27 per cent.
68. 19'568 per sec., 9'18 H.P., 87'7 per cent., and 83'06 per cent.
69. 540'576 volts, 9'67 per sec., 94'4 per cent., and 92'0 per cent.
70. 94'64 per cent. dynamo, 94'91 per cent. motor; 14'3 H.P., 90'6 per cent. as dynamo and 90'6 per cent. as motor.
71. 960 per min., 96'15 per cent. as dynamo, 12'35 H.P., 96 per cent., and 92'2 per cent.
72. 1080 volts, 13'4 H.P., 96'29 per cent., and 92'59 per cent.
73. 79'8 volts, 19'68 H.P., 94'3 per cent., and 90'2 per cent.
74. 53'84 amps., 24'7 per sec., 38'93 H.P., 98'83 per cent., 89'9 per cent., and 78'3 per cent.

75. 367·35 lbs., 28·5 H.P., 94·6 per cent., and 91·3 per cent.
76. 10·56 per sec., 111·07 amps., 97·7 per cent., 95·94 per cent., and 81·549 per cent.
77. 200 wires; R_s 39·8 ohms; R_a 0·0467, R_m 0·0156 ohm.
78. 8·65 H.P., 15·6 per sec., 90·0 per cent., and 86·0 per cent.
79. 0·00054 sq. in. area.
80. 6650.
81. 0·0419 in. diameter.
82. 1,962,770.
83. 5·49 H.P. in res., and 14·7 H.P. in frictions.
84. 2337.
85. 96·6 per cent.
86. 0·0125 ohm, and 18·9 ohms.
87. 1·56 H.P.
88. 96·3 per cent., 50·1 H.P., 7·88 H.P.
89. Dynamo 21·18 per sec., motor 17·18 per sec., 10·71 H.P., 71·4 per cent.
90. 19·29 H.P., dynamo 27·58 per sec., motor 23·58 per sec.
91. 1043·5 volts.
92. 7500.
93. 46 ohms.
94. Shunt wire 0·066 in. diameter, series 30 convolutions, 92·8 per cent.
95. 23·08° C.
96. 148·9 amps.
97. 0·057 in. diameter.
98. No. 15 S.W.G.
99. 96 convolutions.
100. 115·24 volts.
101. 730·2 per min.
102. 0·0081 sq. in.
103. 0·00224 in. diameter; 2·14 watts.
104. 2·2 watts.
105. 0·0143 in. diameter, 11·5 watts.
106. (a) £1 nearly, (b) £2 10s. 0d.
107. 38,000 ohms; 0·0199 per cent.
108. 92·72 in station; 111·11 volts outside.
109. 300 ohms.
110. 30·0388 volts total.
111. 0·005 in. diameter.
112. 0·0031 in. diameter.
113. 0·0056 in. diameter.
114. 0·0112 in. diameter; 1800 ohms.
115. Level, 300 amps., 200 H.P.; up 1 in 480, 356 amps., 237·3 H.P.; up 1 in 384, 370 amps., 246·6 H.P.; down 1 in 960, 272 amps., 181·3 H.P.; up 1 in 960, 328 amps., 218·6 H.P.; down 1 in 384, 230 amps., 153·3 H.P., down 1 in 480, 244 amps., 162·6 H.P.

116. Level 150 amps., 80 H.P. : up 1 in 480, 173·3 amps., 92·4 H.P. ; up 1 in 384, 179·1 amps., 95·52 H.P. ; down 1 in 960, 138·3 amps., 73·76 H.P. ; up 1 in 960, 161·6 amps., 86·16 H.P. ; down 1 in 384, 120·8 amps., 64·4 H.P. ; down 1 in 480, 126·6 amps. ; 67·52 H.P. The E.M.F. in all cases being 397·8 volts.

117. 596·8 amps., 1042 amps., 151·1 amps.

118. 1732 amps.

119. Level 59·68 amps., up 1 in 200, 86·53 amps., down 1 in 200, 32·8 amps., up 1 in 80, 126·5 amps., down 1 in 80, put on brake.

120. 82 amps.

121. 52·4 amps., 5·589 H.P.

122. Level, empty 18·75 amps., loaded 21·875 amps., 2·4 H.P. and 2·8 H.P., up 1 in 50, 65·43 amps., and 8·376 H.P. ; down 1 in 100, 2·193 amps., and 0·28 H.P.

123. Level 8·7 amps., 1504 ft. per min., 3·41 H.P. ; up 1 in 150, 12·3 amps., 1317 ft. per min., 4·78 H.P. ; down 1 in 150, 2·7 amps., 1426 ft. per min., and 1·077 H.P.

124. 115 amps. ; 24·4 miles per hour, 6·71 revolutions per sec.

125. 177 amps., 23·56 miles per hour, 6·5 revolutions per sec.

126. 64 amps., 31·45 miles per hour, 8·69 revolutions per sec.

127. 3·05 mins.

128. 54·2 amps., 2165 ft. per min., 19·6 revolutions per sec.

129. 77 amps., 2273 ft. per min., 20·53 revolutions per sec.

130. About 42 amps.

131. 17·76 amps., 1993 volts.

132. 99 volts.

133. 2·1 amps.

134. 13·4 H.P.

135. 60 volts.

136. x 60 volts, y 31·65 volts.

137. x 84,375, y 27,630.

138. 2020·5 volts.

139. 99·24 volts.

140. 1022·2 volts.

141. 1968, 2021·55 and 2075·1 volts.

142. 95·51 per cent.

143. 92·4, 95·7, 97·3, and 97·5 per cent.

144. 93·6 per cent.

145. 95·5, 93·9, 78·0, and all-day 86·7 per cent.

146. 97·38, 94·7, 88·41, and 79·36 per cent.

147. 2020 volts, 1·25 microfarad.

148. 13½ ohms starting, 5·4 ohms regulating, 6 H.P. slow, and 24·1 H.P. full speed.

149. 1·9 H.P. and 14·3 H.P.

150. 0·252 ohm.

151. 2·21 pence.

152. 1·35 pence.

SPECIFIC RESISTANCE OF VARIOUS MATERIALS AT 0° C., WITH
TEMPERATURE VARIATIONS.

| Material. | Ohms per cubic inch. | Ohms per cubic centimetre. | Temperature variation of resistance per cent. per degree Centigrade. | Weight in pounds per cubic inch. |
|--|-------------------------|-------------------------------|--|--|
| Copper | 0'00000063 | 0'0000016 | 0'388 increase | 0'320 |
| Iron | 0'00000405 | 0'0000104 | 0'453 ,, | 0'280 |
| Aluminium | 0'00000114 | 0'0000029 | 0'390 ,, | 0'092 |
| Mercury | 0'00003666 | 0'0000943 | 0'072 ,, | 0'489 |
| Lead | 0'00000780 | 0'0000200 | 0'387 ,, | 0'410 |
| Platinum | 0'00000353 | 0'0000089 | 0'247 ,, | 0'828 |
| Platinoid | 0'000014 | 0'000034 | 0'021 ,, | |
| German silver | 0'0000082 | 0'000021 | 0'044 ,, | 0'30 |
| Manganin | 0'000018 | 0'000044 | 0'002 decrease | |
| Brass | 0'0000028 | 0'0000072 | | 0'290 |
| Platinum-silver | 0'0000095 | 0'000024 | 0'031 increase | |
| Carbon (arc lamp) { | 0'0017 to 0'0034 | 0'0044 to 0'0086 | } 0'052 decrease | |
| Sulphuric acid— solution 1'1 sp. g. | 3'33 | 8'45 | 2'2 ,, | |
| „ 1'2 sp. g. | 2'62 | 6'66 | 2'2 ,, | |

VALUES OF THE SPECIFIC RESISTANCE OF COMMERCIAL COPPER AT
VARIOUS TEMPERATURES.

| Temperature degrees. | | Resistance of one cubic centimetre in ohms. | Resistance of one cubic inch in ohms. |
|----------------------|-------|--|--|
| Cent. | Fahr. | | |
| 0 | 32 | 0'00000160 | 0'00000063 |
| 5 | 41 | 0'00000163 | 0'00000064 |
| 10 | 50 | 0'00000167 | 0'00000065 |
| 15 | 59 | 0'00000169 | 0'00000067 |
| 20 | 68 | 0'00000172 | 0'00000068 |
| 25 | 77 | 0'00000176 | 0'00000069 |
| 30 | 86 | 0'00000179 | 0'00000071 |
| 35 | 95 | 0'00000182 | 0'00000072 |
| 40 | 104 | 0'00000186 | 0'00000073 |
| 45 | 113 | 0'00000189 | 0'00000074 |
| 50 | 122 | 0'00000193 | 0'00000076 |
| 55 | 131 | 0'00000196 | 0'00000077 |
| 60 | 140 | 0'00000200 | 0'00000078 |
| 65 | 149 | 0'00000203 | 0'00000079 |
| 70 | 158 | 0'00000207 | 0'00000081 |
| 75 | 167 | 0'00000210 | 0'00000082 |
| 80 | 176 | 0'00000214 | 0'00000084 |
| 85 | 185 | 0'00000217 | 0'00000085 |
| 90 | 194 | 0'00000221 | 0'00000087 |
| 95 | 203 | 0'00000224 | 0'00000088 |
| 100 | 212 | 0'00000228 | 0'00000090 |

VALUES OF H CORRESPONDING TO VARIOUS VALUES OF β FOR CYCLIC CHANGES IN β AS A SINE-FUNCTION OF THE TIME, AND HAVING MAXIMA OF 1000, 2000, etc., to 7000.

The values are given for each hundred β for a complete half-period. Values for the other half-period can be easily put in by reversing the signs.

The hysteresis loss h in ergs per cc. per cycle corresponding with these values of the cyclic βH curve are given in the table of values of β and H on pp. 214-218.

| β | H | β | H | β | H | β | H |
|---------|------|---------|------|---------|--------|---------|--------|
| 0 | 0.49 | 600 | 0.72 | 900 | 0.61 | 400 | - 0.20 |
| 100 | 0.55 | 700 | 0.75 | 800 | 0.42 | 300 | - 0.28 |
| 200 | 0.59 | 800 | 0.78 | 700 | 0.23 | 200 | - 0.36 |
| 300 | 0.62 | 900 | 0.80 | 600 | 0.07 | 100 | - 0.42 |
| 400 | 0.66 | 1000 | 0.83 | 500 | - 0.07 | 0 | - 0.49 |
| 500 | 0.69 | | | | | | |
| <hr/> | | | | | | | |
| 0 | 0.62 | 1100 | 0.88 | 1900 | 0.50 | 900 | - 0.28 |
| 100 | 0.65 | 1200 | 0.90 | 1800 | 0.72 | 800 | - 0.34 |
| 200 | 0.67 | 1300 | 0.92 | 1700 | 0.55 | 700 | - 0.38 |
| 300 | 0.69 | 1400 | 0.95 | 1600 | 0.40 | 600 | - 0.42 |
| 400 | 0.72 | 1500 | 0.97 | 1500 | 0.26 | 500 | - 0.46 |
| 500 | 0.74 | 1600 | 1.00 | 1400 | 0.16 | 400 | - 0.50 |
| 600 | 0.76 | 1700 | 1.03 | 1300 | 0.05 | 300 | - 0.53 |
| 700 | 0.78 | 1800 | 1.05 | 1200 | - 0.04 | 200 | - 0.56 |
| 800 | 0.80 | 1900 | 1.08 | 1100 | - 0.14 | 100 | - 0.59 |
| 900 | 0.83 | 2000 | 1.12 | 1000 | - 0.21 | 0 | - 0.62 |
| 1000 | 0.85 | | | | | | |
| <hr/> | | | | | | | |
| 0 | 0.83 | 1600 | 1.09 | 2900 | 1.06 | 1400 | - 0.45 |
| 100 | 0.85 | 1700 | 1.11 | 2800 | 0.80 | 1300 | - 0.49 |
| 200 | 0.87 | 1800 | 1.13 | 2700 | 0.61 | 1200 | - 0.53 |
| 300 | 0.88 | 1900 | 1.14 | 2600 | 0.46 | 1100 | - 0.56 |
| 400 | 0.90 | 2000 | 1.16 | 2500 | 0.33 | 1000 | - 0.60 |
| 500 | 0.91 | 2100 | 1.18 | 2400 | 0.21 | 900 | - 0.63 |
| 600 | 0.92 | 2200 | 1.20 | 2300 | 0.11 | 800 | - 0.66 |
| 700 | 0.94 | 2300 | 1.22 | 2200 | 0.0 | 700 | - 0.68 |
| 800 | 0.96 | 2400 | 1.24 | 2100 | - 0.08 | 600 | - 0.71 |
| 900 | 0.97 | 2500 | 1.26 | 2000 | - 0.15 | 500 | - 0.74 |
| 1000 | 0.99 | 2600 | 1.27 | 1900 | - 0.22 | 400 | - 0.76 |
| 1100 | 1.01 | 2700 | 1.29 | 1800 | - 0.27 | 300 | - 0.79 |
| 1200 | 1.02 | 2800 | 1.31 | 1700 | - 0.32 | 200 | - 0.81 |
| 1300 | 1.04 | 2900 | 1.33 | 1600 | - 0.37 | 100 | - 0.82 |
| 1400 | 1.06 | 3000 | 1.35 | 1500 | - 0.41 | 0 | - 0.83 |
| 1500 | 1.07 | | | | | | |

| β | H | β | H | β | H | β | H |
|---------|------|---------|------|---------|--------|---------|--------|
| 0 | 0'91 | 2100 | 1'20 | 3900 | 1'25 | 1900 | - 0'55 |
| 100 | 0'92 | 2200 | 1'22 | 3800 | 1'00 | 1800 | - 0'58 |
| 200 | 0'94 | 2300 | 1'23 | 3700 | 0'80 | 1700 | - 0'60 |
| 300 | 0'95 | 2400 | 1'25 | 3600 | 0'65 | 1600 | - 0'62 |
| 400 | 0'96 | 2500 | 1'26 | 3500 | 0'52 | 1500 | - 0'65 |
| 500 | 0'98 | 2600 | 1'28 | 3400 | 0'39 | 1400 | - 0'67 |
| 600 | 0'99 | 2700 | 1'30 | 3300 | 0'27 | 1300 | - 0'70 |
| 700 | 1'00 | 2800 | 1'31 | 3200 | 0'17 | 1200 | - 0'72 |
| 800 | 1'01 | 2900 | 1'33 | 3100 | 0'08 | 1100 | - 0'74 |
| 900 | 1'02 | 3000 | 1'35 | 3000 | 0'00 | 1000 | - 0'76 |
| 1000 | 1'03 | 3100 | 1'37 | 2900 | - 0'07 | 900 | - 0'78 |
| 1100 | 1'05 | 3200 | 1'38 | 2800 | - 0'14 | 800 | - 0'79 |
| 1200 | 1'07 | 3300 | 1'40 | 2700 | - 0'20 | 700 | - 0'81 |
| 1300 | 1'09 | 3400 | 1'42 | 2600 | - 0'25 | 600 | - 0'83 |
| 1400 | 1'10 | 3500 | 1'44 | 2500 | - 0'31 | 500 | - 0'84 |
| 1500 | 1'11 | 3600 | 1'46 | 2400 | - 0'36 | 400 | - 0'85 |
| 1600 | 1'12 | 3700 | 1'48 | 2300 | - 0'40 | 300 | - 0'86 |
| 1700 | 1'14 | 3800 | 1'50 | 2200 | - 0'44 | 200 | - 0'88 |
| 1800 | 1'15 | 3900 | 1'53 | 2100 | - 0'47 | 100 | - 0'90 |
| 1900 | 1'17 | 4000 | 1'56 | 2000 | - 0'51 | 0 | - 0'91 |
| 2000 | 1'18 | | | | | | |

| | | | | | | | |
|------|------|------|------|------|--------|------|--------|
| 0 | 0'98 | 2600 | 1'27 | 4900 | 1'34 | 2400 | - 0'68 |
| 100 | 0'99 | 2700 | 1'29 | 4800 | 1'08 | 2300 | - 0'70 |
| 200 | 1'00 | 2800 | 1'31 | 4700 | 0'90 | 2200 | - 0'72 |
| 300 | 1'01 | 2900 | 1'33 | 4600 | 0'74 | 2100 | - 0'74 |
| 400 | 1'02 | 3000 | 1'35 | 4500 | 0'60 | 2000 | - 0'75 |
| 500 | 1'02 | 3100 | 1'37 | 4400 | 0'46 | 1900 | - 0'77 |
| 600 | 1'03 | 3200 | 1'38 | 4300 | 0'34 | 1800 | - 0'78 |
| 700 | 1'04 | 3300 | 1'40 | 4200 | 0'22 | 1700 | - 0'80 |
| 800 | 1'05 | 3400 | 1'42 | 4100 | 0'12 | 1600 | - 0'81 |
| 900 | 1'06 | 3500 | 1'44 | 4000 | 0'00 | 1500 | - 0'82 |
| 1000 | 1'07 | 3600 | 1'46 | 3900 | - 0'08 | 1400 | - 0'83 |
| 1100 | 1'08 | 3700 | 1'48 | 3800 | - 0'17 | 1300 | - 0'84 |
| 1200 | 1'10 | 3800 | 1'50 | 3700 | - 0'24 | 1200 | - 0'85 |
| 1300 | 1'11 | 3900 | 1'53 | 3600 | - 0'30 | 1100 | - 0'86 |
| 1400 | 1'12 | 4000 | 1'55 | 3500 | - 0'36 | 1000 | - 0'88 |
| 1500 | 1'13 | 4100 | 1'57 | 3400 | - 0'40 | 900 | - 0'89 |
| 1600 | 1'14 | 4200 | 1'59 | 3300 | - 0'44 | 800 | - 0'90 |
| 1700 | 1'15 | 4300 | 1'61 | 3200 | - 0'47 | 700 | - 0'91 |
| 1800 | 1'17 | 4400 | 1'63 | 3100 | - 0'50 | 600 | - 0'92 |
| 1900 | 1'18 | 4500 | 1'65 | 3000 | - 0'54 | 500 | - 0'93 |
| 2000 | 1'20 | 4600 | 1'67 | 2900 | - 0'57 | 400 | - 0'94 |
| 2100 | 1'21 | 4700 | 1'70 | 2800 | - 0'59 | 300 | - 0'95 |
| 2200 | 1'23 | 4800 | 1'72 | 2700 | - 0'62 | 200 | - 0'96 |
| 2300 | 1'24 | 4900 | 1'74 | 2600 | - 0'64 | 100 | - 0'97 |
| 2400 | 1'25 | 5000 | 1'76 | 2500 | - 0'66 | 0 | - 0'98 |
| 2500 | 1'26 | | | | | | |

CYCLIC β H VALUES.

| β | H | β | H | β | H | β | H |
|---------|-------|---------|-------|---------|--------|---------|---------|
| 0 | 1'06 | 3100 | 1'42 | 5900 | 1'72 | 2900 | - 0'75 |
| 100 | 1'07 | 3200 | 1'43 | 5800 | 1'40 | 2800 | - 0'76 |
| 200 | 1'08 | 3300 | 1'45 | 5700 | 1'16 | 2700 | - 0'77 |
| 300 | 1'09 | 3400 | 1'46 | 5600 | 0'94 | 2600 | - 0'79 |
| 400 | 1'10 | 3500 | 1'48 | 5500 | 0'78 | 2500 | - 0'80 |
| 500 | 1'11 | 3600 | 1'50 | 5400 | 0'62 | 2400 | - 0'81 |
| 600 | 1'12 | 3700 | 1'52 | 5300 | 0'47 | 2300 | - 0'82 |
| 700 | 1'13 | 3800 | 1'54 | 5200 | 0'37 | 2200 | - 0'84 |
| 800 | 1'14 | 3900 | 1'56 | 5100 | 0'27 | 2100 | - 0'85 |
| 900 | 1'15 | 4000 | 1'58 | 5000 | 0'17 | 2000 | - 0'86 |
| 1000 | 1'16 | 4100 | 1'60 | 4900 | 0'08 | 1900 | - 0'87 |
| 1100 | 1'17 | 4200 | 1'62 | 4800 | 0'00 | 1800 | - 0'88 |
| 1200 | 1'18 | 4300 | 1'64 | 4700 | - 0'10 | 1700 | - 0'89 |
| 1300 | 1'20 | 4400 | 1'66 | 4600 | - 0'17 | 1600 | - 0'90 |
| 1400 | 1'21 | 4500 | 1'68 | 4500 | - 0'24 | 1500 | - 0'91 |
| 1500 | 1'22 | 4600 | 1'70 | 4400 | - 0'28 | 1400 | - 0'92 |
| 1600 | 1'23 | 4700 | 1'72 | 4300 | - 0'33 | 1300 | - 0'93 |
| 1700 | 1'24 | 4800 | 1'74 | 4200 | - 0'37 | 1200 | - 0'94 |
| 1800 | 1'25 | 4900 | 1'76 | 4100 | - 0'42 | 1100 | - 0'95 |
| 1900 | 1'26 | 5000 | 1'78 | 4000 | - 0'46 | 1000 | - 0'96 |
| 2000 | 1'27 | 5100 | 1'80 | 3900 | - 0'49 | 900 | - 0'97 |
| 2100 | 1'28 | 5200 | 1'83 | 3800 | - 0'52 | 800 | - 0'98 |
| 2200 | 1'29 | 5300 | 1'85 | 3700 | - 0'56 | 700 | - 0'99 |
| 2300 | 1'30 | 5400 | 1'88 | 3600 | - 0'59 | 600 | - 0'100 |
| 2400 | 1'31 | 5500 | 1'91 | 3500 | - 0'62 | 500 | - 0'101 |
| 2500 | 1'32 | 5600 | 2'00 | 3400 | - 0'64 | 400 | - 0'102 |
| 2600 | 1'34 | 5700 | 2'10 | 3300 | - 0'67 | 300 | - 0'103 |
| 2700 | 1'35 | 5800 | 2'20 | 3200 | - 0'69 | 200 | - 0'104 |
| 2800 | 1'36 | 5900 | 2'30 | 3100 | - 0'72 | 100 | - 0'105 |
| 2900 | 1'38 | 6000 | 2'40 | 3000 | - 0'74 | 0 | - 0'106 |
| 3000 | 1'40 | | | | | | |
| 0 | 1'120 | 1800 | 1'296 | 3600 | 1'536 | 5400 | 1'890 |
| 100 | 1'128 | 1900 | 1'308 | 3700 | 1'552 | 5500 | 1'910 |
| 200 | 1'136 | 2000 | 1'320 | 3800 | 1'568 | 5600 | 1'940 |
| 300 | 1'144 | 2100 | 1'332 | 3900 | 1'584 | 5700 | 1'960 |
| 400 | 1'152 | 2200 | 1'344 | 4000 | 1'600 | 5800 | 1'980 |
| 500 | 1'160 | 2300 | 1'356 | 4100 | 1'620 | 5900 | 2'010 |
| 600 | 1'170 | 2400 | 1'368 | 4200 | 1'640 | 6000 | 2'050 |
| 700 | 1'180 | 2500 | 1'380 | 4300 | 1'660 | 6100 | 2'080 |
| 800 | 1'190 | 2600 | 1'394 | 4400 | 1'680 | 6200 | 2'110 |
| 900 | 1'200 | 2700 | 1'408 | 4500 | 1'700 | 6300 | 2'140 |
| 1000 | 1'210 | 2800 | 1'422 | 4600 | 1'718 | 6400 | 2'180 |
| 1100 | 1'220 | 2900 | 1'436 | 4700 | 1'736 | 6500 | 2'210 |
| 1200 | 1'230 | 3000 | 1'450 | 4800 | 1'754 | 6600 | 2'250 |
| 1300 | 1'240 | 3100 | 1'464 | 4900 | 1'772 | 6700 | 2'290 |
| 1400 | 1'250 | 3200 | 1'478 | 5000 | 1'790 | 6800 | 2'340 |
| 1500 | 1'260 | 3300 | 1'492 | 5100 | 1'810 | 6900 | 2'380 |
| 1600 | 1'272 | 3400 | 1'506 | 5200 | 1'830 | 7000 | 2'440 |
| 1700 | 1'284 | 3500 | 1'520 | 5300 | 1'860 | 6900 | 1'960 |

| β | H | β | H | β | H | β | H |
|---------|---------|---------|---------|---------|---------|---------|---------|
| 6800 | 1·600 | 5000 | - 0·460 | 3300 | - 0·830 | 1600 | - 0·972 |
| 6700 | 1·380 | 4900 | - 0·492 | 3200 | - 0·840 | 1500 | - 0·980 |
| 6600 | 1·190 | 4800 | - 0·524 | 3100 | - 0·850 | 1400 | - 0·988 |
| 6500 | 1·000 | 4700 | - 0·556 | 3000 | - 0·860 | 1300 | - 0·996 |
| 6400 | 0·840 | 4600 | - 0·588 | 2900 | - 0·870 | 1200 | - 1·004 |
| 6300 | 0·720 | 4500 | - 0·620 | 2800 | - 0·880 | 1100 | - 1·012 |
| 6200 | 0·590 | 4400 | - 0·644 | 2700 | - 0·890 | 1000 | - 1·020 |
| 6100 | 0·440 | 4300 | - 0·668 | 2600 | - 0·900 | 900 | - 1·030 |
| 6000 | 0·320 | 4200 | - 0·692 | 2500 | - 0·910 | 800 | - 1·040 |
| 5900 | 0·200 | 4100 | - 0·716 | 2400 | - 0·918 | 700 | - 1·050 |
| 5800 | 0·110 | 3000 | - 0·740 | 2300 | - 0·926 | 600 | - 1·060 |
| 5700 | 0·000 | 3900 | - 0·752 | 2200 | - 0·934 | 500 | - 1·070 |
| 5600 | - 0·100 | 3800 | - 0·764 | 2100 | - 0·942 | 400 | - 1·080 |
| 5500 | - 0·200 | 3700 | - 0·776 | 2000 | - 0·950 | 300 | - 1·090 |
| 5400 | - 0·252 | 3600 | - 0·780 | 1900 | - 0·956 | 200 | - 1·100 |
| 5300 | - 0·304 | 3500 | - 0·810 | 1800 | - 0·962 | 100 | - 1·110 |
| 5200 | - 0·356 | 3400 | - 0·820 | 1700 | - 0·968 | 0 | - 1·120 |
| 5100 | - 0·403 | | | | | | |

ANNEALED CHARCOAL IRON. VALUES OF MAGNETIC PROPERTIES.

| β Density in c.g.s. lines per sq. cm. area. | H in c.g.s. per cm. length. | $f\beta$ Ampère-turns per cm. length. | μ Permea- bility. | h Hysteresis loss in ergs per cc. per cycle. | $\beta/f\beta$ Product of density and ampère-turns per cm. length. |
|--|-----------------------------------|--|-----------------------------|--|--|
| 100 | 0'17 | 0'140 | 588 | 5 | 14 |
| 200 | 0'32 | 0'260 | 625 | 10 | 52 |
| 300 | 0'44 | 0'356 | 685 | 17 | 107 |
| 400 | 0'54 | 0'432 | 740 | 25 | 172 |
| 500 | 0'60 | 0'476 | 833 | 37 | 238 |
| 600 | 0'65 | 0'520 | 908 | 50 | 312 |
| 700 | 0'69 | 0'552 | 1014 | 65 | 386 |
| 800 | 0'76 | 0'584 | 1052 | 80 | 467 |
| 900 | 0'78 | 0'624 | 1154 | 97 | 561 |
| 1000 | 0'83 | 0'664 | 1204 | 115 | 664 |
| 1100 | 0'87 | 0'696 | 1264 | 134 | 765 |
| 1200 | 0'91 | 0'728 | 1318 | 155 | 873 |
| 1300 | 0'94 | 0'752 | 1382 | 177 | 978 |
| 1400 | 0'97 | 0'776 | 1443 | 200 | 1086 |
| 1500 | 0'99 | 0'796 | 1515 | 222 | 1194 |
| 1600 | 1'02 | 0'816 | 1568 | 245 | 1305 |
| 1700 | 1'04 | 0'836 | 1634 | 270 | 1421 |
| 1800 | 1'07 | 0'856 | 1682 | 295 | 1540 |
| 1900 | 1'09 | 0'872 | 1743 | 320 | 1656 |
| 2000 | 1'11 | 0'888 | 1802 | 345 | 1776 |
| 2100 | 1'13 | 0'904 | 1858 | 372 | 1898 |
| 2200 | 1'15 | 0'922 | 1913 | 400 | 2024 |
| 2300 | 1'17 | 0'940 | 1966 | 430 | 2162 |
| 2400 | 1'19 | 0'958 | 2117 | 460 | 2304 |
| 2500 | 1'22 | 0'976 | 2049 | 490 | 2440 |
| 2600 | 1'24 | 0'992 | 2096 | 520 | 2579 |
| 2700 | 1'26 | 1'01 | 2142 | 550 | 2727 |
| 2800 | 1'29 | 1'03 | 2170 | 580 | 2889 |
| 2900 | 1'31 | 1'05 | 2214 | 610 | 3045 |
| 3000 | 1'34 | 1'07 | 2238 | 640 | 3216 |
| 3100 | 1'36 | 1'09 | 2279 | 670 | 3379 |
| 3200 | 1'38 | 1'10 | 2318 | 700 | 3520 |
| 3300 | 1'40 | 1'12 | 2357 | 735 | 3696 |
| 3400 | 1'42 | 1'14 | 2394 | 770 | 3862 |
| 3500 | 1'44 | 1'16 | 2430 | 805 | 4065 |
| 3600 | 1'47 | 1'18 | 2448 | 840 | 4233 |
| 3700 | 1'49 | 1'20 | 2486 | 877 | 4440 |
| 3800 | 1'51 | 1'21 | 2516 | 915 | 4590 |
| 3900 | 1'53 | 1'22 | 2548 | 952 | 4758 |

ANNEALED CHARCOAL IRON. VALUES OF MAGNETIC PROPERTIES.

| β Density in c.g.s. lines per sq. cm. area. | H in c.g.s. per cm. length. | $f\beta$ Ampère-turns per cm. length. | μ Permea- bility. | h Hysteresis loss in ergs per cc. per cycle. | $\beta/f\beta$ Product of density and ampère-turns per cm. length. |
|--|-----------------------------------|--|-----------------------------|--|--|
| 4000 | 1'55 | 1'24 | 2580 | 990 | 4960 |
| 4100 | 1'57 | 1'26 | 2611 | 1022 | 5166 |
| 4200 | 1'59 | 1'27 | 2641 | 1055 | 5346 |
| 4300 | 1'61 | 1'29 | 2670 | 1092 | 5567 |
| 4400 | 1'63 | 1'30 | 2699 | 1130 | 5781 |
| 4500 | 1'65 | 1'32 | 2726 | 1170 | 5940 |
| 4600 | 1'67 | 1'34 | 2754 | 1210 | 6145 |
| 4700 | 1'69 | 1'36 | 2780 | 1252 | 6392 |
| 4800 | 1'72 | 1'38 | 2790 | 1295 | 6604 |
| 4900 | 1'74 | 1'39 | 2816 | 1337 | 6811 |
| 5000 | 1'76 | 1'41 | 2841 | 1380 | 7040 |
| 5100 | 1'78 | 1'42 | 2865 | 1420 | 7242 |
| 5200 | 1'80 | 1'44 | 2889 | 1460 | 7488 |
| 5300 | 1'82 | 1'46 | 2912 | 1505 | 7738 |
| 5400 | 1'85 | 1'48 | 2918 | 1550 | 7992 |
| 5500 | 1'88 | 1'50 | 2925 | 1590 | 8250 |
| 5600 | 1'91 | 1'53 | 2931 | 1630 | 8556 |
| 5700 | 1'94 | 1'55 | 2938 | 1677 | 8835 |
| 5800 | 1'97 | 1'58 | 2944 | 1725 | 9140 |
| 5900 | 2'00 | 1'60 | 2950 | 1772 | 9445 |
| 6000 | 2'03 | 1'62 | 2955 | 1820 | 9744 |
| 6100 | 2'06 | 1'65 | 2961 | 1867 | 10065 |
| 6200 | 2'10 | 1'68 | 2955 | 1915 | 10416 |
| 6300 | 2'13 | 1'71 | 2952 | 1962 | 10773 |
| 6400 | 2'17 | 1'74 | 2949 | 2010 | 11110 |
| 6500 | 2'21 | 1'77 | 2941 | 2060 | 11505 |
| 6600 | 2'25 | 1'80 | 2938 | 2110 | 11880 |
| 6700 | 2'28 | 1'83 | 2933 | 2160 | 12261 |
| 6800 | 2'32 | 1'86 | 2930 | 2210 | 12662 |
| 6900 | 2'36 | 1'89 | 2923 | 2260 | 13041 |
| 7000 | 2'41 | 1'93 | 2904 | 2310 | 13496 |
| 7100 | 2'46 | 1'97 | 2886 | 2362 | 13987 |
| 7200 | 2'51 | 2'01 | 2868 | 2415 | 14457 |
| 7300 | 2'56 | 2'05 | 2851 | 2470 | 14965 |
| 7400 | 2'62 | 2'10 | 2824 | 2525 | 15510 |
| 7500 | 2'67 | 2'14 | 2808 | 2587 | 16050 |
| 7600 | 2'73 | 2'18 | 2783 | 2650 | 16598 |
| 7700 | 2'78 | 2'23 | 2769 | 2705 | 17171 |
| 7800 | 2'84 | 2'27 | 2746 | 2760 | 17721 |
| 7900 | 2'90 | 2'32 | 2723 | 2810 | 18328 |
| 8000 | 2'96 | 2'37 | 2703 | 2860 | 18944 |
| 8100 | 3'01 | 2'41 | 2691 | 2939 | 19521 |
| 8200 | 3'06 | 2'45 | 2679 | 3018 | 20090 |
| 8300 | 3'12 | 2'50 | 2660 | 3097 | 20750 |
| 8400 | 3'18 | 2'55 | 2641 | 3176 | 21420 |

ANNEALED CHARCOAL IRON. VALUES OF MAGNETIC PROPERTIES.

| β Density in c.g.s. lines per sq. cm. area. | H in c.g.s. per cm. length. | $f\beta$ Ampère-turns per cm. length. | μ Permea- bility. | h Hysteresis loss in ergs per cc. per cycle. | $\beta\beta$ Product of density and ampère-turns per cm. length. |
|--|-----------------------------------|--|-----------------------------|--|--|
| 8500 | 3'25 | 2'60 | 2615 | 3255 | 22100 |
| 8600 | 3'31 | 2'65 | 2598 | 3334 | 22790 |
| 8700 | 3'37 | 2'70 | 2581 | 3413 | 23490 |
| 8800 | 3'43 | 2'75 | 2565 | 3492 | 24200 |
| 8900 | 3'50 | 2'80 | 2542 | 3571 | 24720 |
| 9000 | 3'56 | 2'85 | 2528 | 3650 | 25650 |
| 9100 | 3'62 | 2'90 | 2513 | 3730 | 26390 |
| 9200 | 3'68 | 2'95 | 2500 | 3810 | 27140 |
| 9300 | 3'76 | 3'01 | 2473 | 3890 | 27993 |
| 9400 | 3'85 | 3'08 | 2441 | 3970 | 28952 |
| 9500 | 3'92 | 3'14 | 2423 | 4050 | 29830 |
| 9600 | 4'00 | 3'20 | 2400 | 4130 | 30730 |
| 9700 | 4'08 | 3'27 | 2377 | 4210 | 31719 |
| 9800 | 4'16 | 3'35 | 2355 | 4290 | 32830 |
| 9900 | 4'27 | 3'42 | 2318 | 4370 | 33858 |
| 10000 | 4'37 | 3'50 | 2288 | 4450 | 35000 |
| 10100 | 4'48 | 3'59 | 2254 | 4535 | 36259 |
| 10200 | 4'60 | 3'68 | 2217 | 4620 | 37536 |
| 10300 | 4'71 | 3'77 | 2187 | 4705 | 38831 |
| 10400 | 4'82 | 3'86 | 2157 | 4790 | 40147 |
| 10500 | 4'96 | 3'96 | 2117 | 4875 | 41580 |
| 10600 | 5'07 | 4'06 | 2090 | 4960 | 43036 |
| 10700 | 5'20 | 4'16 | 2057 | 5045 | 44512 |
| 10800 | 5'33 | 4'27 | 2026 | 5130 | 46116 |
| 10900 | 5'46 | 4'37 | 1995 | 5215 | 47633 |
| 11000 | 5'60 | 4'48 | 1963 | 5300 | 49280 |
| 11100 | 5'73 | 4'59 | 1937 | 5390 | 50949 |
| 11200 | 5'87 | 4'70 | 1908 | 5480 | 52640 |
| 11300 | 6'03 | 4'83 | 1873 | 5570 | 54579 |
| 11400 | 6'20 | 4'96 | 1838 | 5650 | 56544 |
| 11500 | 6'40 | 5'12 | 1797 | 5750 | 58880 |
| 11600 | 6'60 | 5'28 | 1757 | 5840 | 61248 |
| 11700 | 6'85 | 5'48 | 1707 | 5930 | 64116 |
| 11800 | 7'10 | 5'68 | 1662 | 6020 | 67024 |
| 11900 | 7'36 | 5'88 | 1617 | 6110 | 69972 |
| 12000 | 7'60 | 6'08 | 1579 | 6200 | 72960 |
| 12100 | 7'90 | 6'32 | 1531 | 6300 | 76472 |
| 12200 | 8'20 | 6'56 | 1487 | 6400 | 80032 |
| 12300 | 8'50 | 6'80 | 1437 | 6500 | 83640 |
| 12400 | 8'90 | 7'12 | 1393 | 6600 | 88288 |
| 12500 | 9'20 | 7'36 | 1351 | 6700 | 92000 |
| 12600 | 9'60 | 7'68 | 1312 | 6800 | 96768 |
| 12700 | 10'0 | 8'00 | 1270 | 6900 | 101600 |
| 12800 | 10'4 | 8'32 | 1231 | 7000 | 106496 |
| 12900 | 10'8 | 8'64 | 1192 | 7100 | 111456 |

ANNEALED CHARCOAL IRON. VALUES OF MAGNETIC PROPERTIES.

| β Density in c.g.s. lines per sq. cm. area. | H in c.g.s. per cm. length. | $f\beta$ Ampère-turns per c.m. length. | μ Permea- bility. | h Hysteresis loss in ergs per cc. per cycle. | $\beta/f\beta$ Product of density and ampère-turns per cm. length. |
|--|-----------------------------------|---|-----------------------------|--|--|
| 13000 | 11'3 | 9'04 | 1151 | 7200 | 117520 |
| 13100 | 11'7 | 9'36 | 1115 | 7300 | 122616 |
| 13200 | 12'2 | 9'76 | 1082 | 7400 | 128820 |
| 13300 | 12'7 | 10'1 | 1047 | 7500 | 134330 |
| 13400 | 13'2 | 10'5 | 1015 | 7600 | 140700 |
| 13500 | 13'7 | 10'9 | 986 | 7700 | 147150 |
| 13600 | 14'2 | 11'3 | 958 | 7800 | 153680 |
| 13700 | 14'8 | 11'8 | 926 | 7900 | 161600 |
| 13800 | 15'4 | 12'3 | 896 | 8000 | 169740 |
| 13900 | 16'1 | 12'8 | 863 | 8100 | 177920 |
| 14000 | 16'9 | 13'5 | 839 | 8200 | 189000 |
| 14100 | 17'8 | 14'2 | 793 | 8310 | 200220 |
| 14200 | 18'7 | 14'9 | 759 | 8420 | 211580 |
| 14300 | 19'7 | 15'7 | 725 | 8530 | 224510 |
| 14400 | 20'7 | 16'5 | 695 | 8640 | 237600 |
| 14500 | 21'8 | 17'4 | 665 | 8750 | 252300 |
| 14600 | 23'0 | 18'4 | 636 | 8860 | 278640 |
| 14700 | 24'2 | 19'3 | 608 | 8970 | 283710 |
| 14800 | 25'5 | 20'4 | 580 | 9080 | 301920 |
| 14900 | 26'8 | 21'4 | 555 | 9190 | 318860 |
| 15000 | 28'6 | 22'8 | 524 | 9300 | 340000 |
| 15100 | 30'0 | 24'0 | 503 | 9410 | 362400 |
| 15200 | 31'5 | 25'2 | 482 | 9520 | 383040 |
| 15300 | 33'1 | 26'4 | 461 | 9630 | 403920 |
| 15400 | 35'0 | 28'0 | 440 | 9740 | 431200 |
| 15500 | 37'1 | 29'6 | 419 | 9850 | 458800 |
| 15600 | 39'2 | 31'3 | 398 | 9960 | 488280 |
| 15700 | 41'5 | 33'2 | 379 | 10070 | 521240 |
| 15800 | 43'8 | 35'0 | 361 | 10180 | 553000 |
| 15900 | 46'8 | 37'4 | 341 | 10290 | 594660 |
| 16000 | 48'8 | 39'0 | 321 | 10400 | 624000 |
| 16100 | 53'2 | 42'5 | 303 | 10510 | 684250 |
| 16200 | 56'7 | 45'3 | 285 | 10620 | 733860 |
| 16300 | 60'7 | 48'5 | 270 | 10730 | 790550 |
| 16400 | 65'0 | 52'0 | 256 | 10840 | 852800 |
| 16500 | 69'9 | 55'9 | 238 | 10950 | 922340 |
| 16600 | 75'2 | 60'1 | 221 | 11060 | 997660 |
| 16700 | 80'7 | 64'5 | 207 | 11170 | 1077150 |
| 16800 | 87'0 | 69'6 | 193 | 11280 | 1169280 |
| 16900 | 93'0 | 74'4 | 181 | 11390 | 1257360 |
| 17000 | 100'0 | 80'0 | 170 | 11500 | 1360000 |
| 17100 | 107'0 | 85'5 | 159 | 11620 | 1462050 |
| 17200 | 115'0 | 92'0 | 149 | 11740 | 1582400 |
| 17300 | 123'0 | 98'4 | 141 | 11860 | 1702320 |
| 17400 | 131'0 | 104'0 | 133 | 11980 | 1809600 |

ANNEALED CHARCOAL IRON. VALUES OF MAGNETIC PROPERTIES.

| β Density in c.g.s. lines per sq. cm. area. | H in c.g.s. per cm. length. | $f\beta$ Ampère-turns per cm. length. | μ Permea- bility. | h Hysteresis loss in ergo per cc. per cycle. | $\beta f\beta$ Product of density and ampère-turns per cm. length. |
|--|-----------------------------------|--|-----------------------------|--|--|
| 17500 | 140 | 112 | 125 | 12100 | 1960000 |
| 17600 | 149 | 119 | 117 | 12220 | 2094400 |
| 17700 | 158 | 126 | 111 | 12340 | 2230200 |
| 17800 | 167 | 133 | 106 | 12460 | 2367400 |
| 17900 | 176 | 140 | 101 | 12580 | 2506000 |
| 18000 | 186 | 148 | 96 | 12700 | 2664000 |
| 18100 | 196 | 156 | 90 | 12820 | 2823600 |
| 18200 | 211 | 168 | 86 | 12940 | 3057600 |
| 18300 | 226 | 180 | 80 | 13060 | 3294000 |
| 18400 | 244 | 195 | 71 | 13180 | 3588000 |
| 18500 | 260 | 208 | 69 | 13300 | 3848000 |
| 18600 | 276 | 220 | 67 | 13420 | 4092000 |
| 18700 | 293 | 234 | 65 | 13540 | 4375800 |
| 18800 | 311 | 248 | 64 | 13660 | 4662400 |
| 18900 | 330 | 264 | 60 | 13780 | 4989600 |
| 19000 | 350 | 280 | 54 | 13900 | 5320000 |
| 19100 | 372 | 297 | 51 | 14020 | 5672700 |
| 19200 | 396 | 316 | 48 | 14140 | 6067200 |
| 19300 | 422 | 337 | 45 | 14260 | 6504100 |
| 19400 | 448 | 358 | 43 | 14380 | 6945200 |
| 19500 | 476 | 380 | 41 | 14500 | 7410000 |
| 19600 | 505 | 404 | 39 | 14620 | 7918400 |
| 19700 | 537 | 429 | 36 | 14740 | 8451300 |
| 19800 | 575 | 460 | 34 | 14860 | 9108000 |
| 19900 | 609 | 487 | 32 | 14980 | 9691300 |
| 20000 | 650 | 520 | 31 | 15100 | 10400000 |

MAGNETIC PROPERTIES OF SOFT GREY CAST IRON.

| β Density in c.g.s. lines per sq. cm. | H c.g.s. per cm. length. | $f\beta$ Ampère-turns per cm. length. | μ Permeability. |
|---|--------------------------------|---|------------------------|
| 3000 | 1'5 | 1'2 | 2000 |
| 3100 | 1'9 | 1'5 | 1695 |
| 3200 | 2'3 | 1'8 | 1391 |
| 3300 | 2'7 | 2'2 | 1241 |
| 3400 | 3'1 | 2'5 | 1096 |
| 3500 | 3'5 | 2'8 | 1009 |
| 3600 | 3'9 | 3'1 | 923 |
| 3700 | 4'3 | 3'4 | 860 |
| 3800 | 4'7 | 3'8 | 808 |
| 3900 | 5'1 | 4'1 | 764 |
| 4000 | 5'5 | 4'4 | 727 |
| 4100 | 5'9 | 4'7 | 695 |
| 4200 | 6'3 | 5'0 | 666 |
| 4300 | 6'7 | 5'4 | 641 |
| 4400 | 7'1 | 5'7 | 619 |
| 4500 | 7'6 | 6'1 | 592 |
| 4600 | 8'1 | 6'5 | 568 |
| 4700 | 8'7 | 7'0 | 540 |
| 4800 | 9'3 | 7'4 | 516 |
| 4900 | 10'0 | 8'0 | 490 |
| 5000 | 10'7 | 8'6 | 467 |
| 5100 | 11'6 | 9'3 | 439 |
| 5200 | 12'5 | 10'0 | 416 |
| 5300 | 13'5 | 10'8 | 392 |
| 5400 | 14'5 | 11'6 | 372 |
| 5500 | 15'8 | 12'6 | 348 |
| 5600 | 17'1 | 13'7 | 327 |
| 5700 | 18'5 | 14'8 | 307 |
| 5800 | 20'0 | 16'0 | 290 |
| 5900 | 21'6 | 17'3 | 274 |
| 6000 | 23'2 | 18'5 | 258 |
| 6100 | 24'8 | 19'8 | 246 |
| 6200 | 26'5 | 21'2 | 234 |
| 6300 | 28'2 | 22'5 | 223 |
| 6400 | 30'0 | 24'0 | 213 |
| 6500 | 32'0 | 25'6 | 203 |
| 6600 | 34'0 | 27'2 | 194 |
| 6700 | 36'4 | 29'1 | 186 |
| 6800 | 38'4 | 30'7 | 178 |
| 6900 | 40'8 | 32'6 | 174 |
| 7000 | 43'2 | 34'6 | 162 |

MAGNETIC PROPERTIES OF SOFT GREY CAST IRON.

| β Density in c.g.s. lines per sq. cm. | H c.g.s. per cm. length. | $f\beta$ Ampère-turns per cm. length. | μ Permeability. |
|---|--------------------------------|---|------------------------|
| 7100 | 45.9 | 36.7 | 155 |
| 7200 | 48.6 | 38.9 | 148 |
| 7300 | 51.7 | 41.4 | 141 |
| 7400 | 54.9 | 43.9 | 135 |
| 7500 | 58.6 | 46.8 | 129 |
| 7600 | 62.3 | 49.8 | 123 |
| 7700 | 67.2 | 53.7 | 117 |
| 7800 | 70.2 | 56.1 | 111 |
| 7900 | 74.3 | 59.4 | 106 |
| 8000 | 78.5 | 62.8 | 102 |
| 8100 | 82.7 | 66.1 | 98 |
| 8200 | 86.9 | 69.5 | 94 |
| 8300 | 91.1 | 72.8 | 91 |
| 8400 | 95.4 | 76.3 | 89 |
| 8500 | 99.8 | 79.8 | 86 |
| 8600 | 104.2 | 83.3 | 83 |
| 8700 | 108.9 | 87.1 | 80 |
| 8800 | 113.6 | 90.8 | 77 |
| 8900 | 118.6 | 94.8 | 74 |
| 9000 | 123.7 | 98.9 | 72 |
| 9100 | 129.5 | 103.6 | 70 |
| 9200 | 135.6 | 108.4 | 68 |
| 9300 | 141.9 | 113.5 | 66 |
| 9400 | 148.3 | 118.6 | 64 |
| 9500 | 154.8 | 123.8 | 61 |
| 9600 | 161.4 | 129.1 | 59 |
| 9700 | 168.0 | 134.4 | 57 |
| 9800 | 174.7 | 139.7 | 55 |
| 9900 | 181.5 | 145.2 | 54 |
| 10000 | 188.5 | 150.8 | 53 |
| 10100 | 195.8 | 156.6 | 51 |
| 10200 | 203.2 | 162.5 | 49 |
| 10300 | 210.8 | 168.6 | 48 |
| 10400 | 219.0 | 175.2 | 47 |
| 10500 | 228.0 | 182.4 | 45 |
| 10600 | 238.0 | 190.4 | 44 |
| 10700 | 248.0 | 198.4 | 43 |
| 10800 | 259.0 | 207.2 | 42 |
| 10900 | 272.0 | 217.6 | 40 |
| 11000 | 288.0 | 230.4 | 39 |

DETAILS OF CONDUCTORS.

| S.W.G. | Diameter of each wire. | Diameter of the strand. | Area. | Resistance at 60° F. | Weight. |
|--------|------------------------------|-------------------------------|----------------|-------------------------|--------------------|
| | Inch. | Inch. | Square inches. | Per 1000 yards. | Per 1000 yards. |
| | | | | Ohms. | lbs. |
| 7/0 | 0'500 | ... | 0'196349 | 0'1246 | 2270 |
| 6/0 | 0'464 | ... | 0'169093 | 0'1446 | 1955 |
| 5/0 | 0'432 | ... | 0'146574 | 0'1669 | 1694 |
| 4/0 | 0'400 | ... | 0'125663 | 0'1946 | 1454 |
| 3/0 | 0'372 | ... | 0'108686 | 0'2251 | 1256 |
| 2/0 | 0'348 | ... | 0'095114 | 0'2572 | 1099 |
| 1/0 | 0'324 | ... | 0'082447 | 0'2967 | 953 |
| 1 | 0'300 | ... | 0'070685 | 0'3461 | 817 |
| 2 | 0'276 | ... | 0'05982 | 0'4089 | 691 |
| 3 | 0'252 | ... | 0'04987 | 0'4905 | 576 |
| 4 | 0'232 | ... | 0'04227 | 0'5787 | 488 |
| 5 | 0'212 | ... | 0'03529 | 0'6931 | 405 |
| 6 | 0'192 | ... | 0'0289 | 0'8450 | 334 |
| 7 | 0'176 | ... | 0'0243 | 1'0056 | 281 |
| 8 | 0'160 | ... | 0'0201 | 1'2168 | 232 |
| 9 | 0'144 | ... | 0'0163 | 1'5022 | 188 |
| 10 | 0'128 | ... | 0'0128 | 1'9012 | 148 |
| 11 | 0'116 | ... | 0'0105 | 2'3150 | 122 |
| 12 | 0'104 | ... | 0'0085 | 2'8800 | 98 |
| 13 | 0'092 | ... | 0'0066 | 3'6803 | 76 |
| 14 | 0'080 | ... | 0'0050 | 4'8673 | 58 |
| 15 | 0'072 | ... | 0'0040 | 6'0089 | 47 |
| 16 | 0'064 | ... | 0'0032 | 7'6049 | 37 |
| 17 | 0'056 | ... | 0'0024 | 9'9332 | 28 |
| 18 | 0'048 | ... | 0'0018 | 13'5198 | 21 |
| 19 | 0'040 | ... | 0'0012 | 19'4697 | 14'5 |
| 20 | 0'036 | ... | 0'0010 | 24'0354 | 11'7 |
| 21 | 0'032 | ... | 0'0008 | 30'422 | 9'3 |
| 22 | 0'028 | ... | 0'0006 | 39'729 | 7'1 |
| 3/25 | 0'020 | 0'042 | 0'00096 | 25'955 | 11 |
| 3/24 | 0'022 | 0'022 | 0'00116 | 21'454 | 13'5 |
| 3/23 | 0'024 | 0'024 | 0'00138 | 18'026 | 16 |
| 3/22 | 0'028 | 0'028 | 0'00188 | 13'243 | 22 |
| 3/21 | 0'032 | 0'032 | 0'00246 | 10'144 | 28 |
| 3/20 | 0'036 | 0'036 | 0'00311 | 8'0118 | 36 |
| 3/19 | 0'040 | 0'040 | 0'00384 | 6'4899 | 44 |
| 3/18 | 0'048 | 0'048 | 0'0055 | 4'5066 | 64 |

DETAILS OF CONDUCTORS.

| S.W.G. | Diameter of each wire. | Diameter of the strand. | Area. | Resistance at 60° F. | Weight. |
|--------|------------------------------|-------------------------------|----------------|-------------------------|--------------------|
| | Inch. | Inch. | Square inches. | Per 1000 yards. | Per 1000 yards. |
| | | | | Ohms. | lbs. |
| 7/25 | 0'020 | 0'060 | 0'0022 | 11'124 | 25'5 |
| 7/24 | 0'022 | 0'066 | 0'0027 | 9'1952 | 31'5 |
| 7/23 | 0'024 | 0'072 | 0'0032 | 7'7256 | 37 |
| 7/22 | 0'028 | 0'084 | 0'0043 | 5'6757 | 51 |
| 7/21½ | 0'030 | 0'090 | 0'0050 | 4'9445 | 58 |
| 7/21 | 0'032 | 0'096 | 0'0057 | 4'3460 | 66 |
| 7/20½ | 0'033 | 0'099 | 0'0064 | 4'0864 | 75 |
| 7/20 | 0'036 | 0'108 | 0'0072 | 3'4336 | 84 |
| 7/19 | 0'040 | 0'120 | 0'0089 | 2'7813 | 104 |
| 7/18 | 0'048 | 0'144 | 0'0129 | 1'9314 | 149 |
| 7/17 | 0'056 | 0'168 | 0'0175 | 1'4190 | 203 |
| 7/16 | 0'064 | 0'192 | 0'0229 | 1'0864 | 266 |
| 7/15 | 0'072 | 0'216 | 0'0290 | 0'8584 | 336 |
| 7/14 | 0'080 | 0'240 | 0'0358 | 0'6953 | 415 |
| 7/13 | 0'092 | 0'276 | 0'0474 | 0'5257 | 549 |
| 7/12 | 0'104 | 0'312 | 0'0606 | 0'4114 | 701 |
| 7/11 | 0'116 | 0'348 | 0'0754 | 0'3307 | 872 |
| 7/10 | 0'128 | 0'384 | 0'0918 | 0'2716 | 1062 |
| 7/9 | 0'144 | 0'432 | 0'1162 | 0'2146 | 1343 |
| 7/8 | 0'160 | 0'480 | 0'1435 | 0'1752 | 1660 |
| 7/6 | 0'192 | 0'576 | 0'2067 | 0'1207 | 2390 |
| 19/24 | 0'022 | 0'110 | 0'0073 | 3'3877 | 85 |
| 19/23 | 0'024 | 0'120 | 0'0087 | 2'8463 | 101'5 |
| 19/22 | 0'028 | 0'140 | 0'0119 | 2'0910 | 138 |
| 19/21 | 0'032 | 0'160 | 0'0156 | 1'6011 | 180 |
| 19/20 | 0'036 | 0'180 | 0'0197 | 1'2650 | 228 |
| 19/19 | 0'040 | 0'200 | 0'0244 | 1'0247 | 282 |
| 19/18 | 0'048 | 0'240 | 0'0351 | 0'7115 | 406 |
| 19/17 | 0'056 | 0'280 | 0'0478 | 0'5228 | 553 |
| 19/16 | 0'064 | 0'320 | 0'0624 | 0'4002 | 722 |
| 19/15 | 0'072 | 0'360 | 0'0790 | 0'3162 | 914 |
| 19/14 | 0'080 | 0'400 | 0'0976 | 0'2561 | 1128 |
| 19/13 | 0'092 | 0'460 | 0'1290 | 0'1937 | 1491 |
| 19/12 | 0'104 | 0'520 | 0'1649 | 0'1515 | 1906 |
| 19/11 | 0'116 | 0'580 | 0'2052 | 0'1218 | 2372 |
| 19/10 | 0'128 | 0'640 | 0'2498 | 0'1000 | 2888 |
| 19/9 | 0'144 | 0'720 | 0'3162 | 0'07906 | 3655 |
| 19/8 | 0'160 | 0'800 | 0'3904 | 0'06406 | 4513 |
| 19/7 | 0'176 | 0'880 | 0'4724 | 0'05292 | 5461 |

DETAILS OF CONDUCTORS.

| S.W.G. | Diameter of each wire. | Diameter of the strand. | Area. | Resistance at 60° F. | Weight. |
|--------|------------------------------|-------------------------------|----------------|-------------------------|--------------------|
| | Inch. | Inch. | Square inches. | Per 1000 yards. | Per 1000 yards. |
| | | | | Ohms. | lbs. |
| 37/24 | 0'022 | 0'154 | 0'0143 | 1'7396 | 165 |
| 37/23 | 0'024 | 0'168 | 0'0171 | 1'4647 | 198 |
| 37/22 | 0'028 | 0'196 | 0'0233 | 1'0737 | 270 |
| 37/21 | 0'032 | 0'224 | 0'0304 | 0'8222 | 352 |
| 37/20 | 0'036 | 0'252 | 0'0386 | 0'6496 | 446 |
| 37/19 | 0'040 | 0'280 | 0'0476 | 0'5262 | 550 |
| 37/18 | 0'048 | 0'336 | 0'0686 | 0'3654 | 793 |
| 37/17 | 0'056 | 0'392 | 0'0934 | 0'2684 | 1080 |
| 37/16 | 0'064 | 0'448 | 0'1220 | 0'2055 | 1410 |
| 37/15 | 0'072 | 0'504 | 0'1544 | 0'1624 | 1785 |
| 37/14 | 0'080 | 0'560 | 0'1906 | 0'1315 | 2203 |
| 37/13 | 0'092 | 0'644 | 0'2521 | 0'09947 | 2914 |
| 37/12 | 0'104 | 0'728 | 0'3221 | 0'07783 | 3723 |
| 37/11 | 0'116 | 0'812 | 0'4008 | 0'06265 | 4633 |
| 37/10 | 0'128 | 0'896 | 0'4880 | 0'05138 | 5641 |
| 37/9 | 0'144 | 1'008 | 0'6176 | 0'04060 | 7140 |
| 37/8 | 0'160 | 1'120 | 0'7625 | 0'03288 | 8815 |
| | | | | | |
| 61/24 | 0'022 | 0'198 | 0'02378 | 1'0551 | 275 |
| 61/23 | 0'024 | 0'216 | 0'02831 | 0'8865 | 327 |
| 61/22 | 0'028 | 0'252 | 0'03854 | 0'6583 | 446 |
| 61/21 | 0'032 | 0'288 | 0'0503 | 0'4987 | 572 |
| 61/20 | 0'036 | 0'324 | 0'0637 | 0'3940 | 736 |
| 61/19 | 0'040 | 0'360 | 0'0786 | 0'3191 | 909 |
| 61/18 | 0'048 | 0'432 | 0'1132 | 0'2216 | 1309 |
| 61/17 | 0'056 | 0'504 | 0'1541 | 0'1628 | 1781 |
| 61/16 | 0'064 | 0'576 | 0'2013 | 0'1246 | 2327 |
| 61/15 | 0'072 | 0'648 | 0'2548 | 0'09850 | 2945 |
| 61/14 | 0'080 | 0'720 | 0'3145 | 0'07979 | 3636 |
| 61/13 | 0'092 | 0'828 | 0'4160 | 0'06033 | 4809 |
| 61/12 | 0'104 | 0'936 | 0'5316 | 0'04721 | 6145 |
| 61/11 | 0'116 | 1'044 | 0'6614 | 0'03795 | 7646 |
| 61/10 | 0'128 | 1'152 | 0'8053 | 0'03116 | 9309 |
| | | | | | |
| 91/18 | 0'048 | 0'528 | 0'1692 | 0'14857 | 1956 |
| 91/17 | 0'056 | 0'616 | 0'2302 | 9'10915 | 2661 |
| 91/16 | 0'064 | 0'704 | 0'3007 | 0'08357 | 3476 |
| 91/15 | 0'072 | 0'792 | 0'3806 | 0'06603 | 4400 |
| 91/14 | 0'080 | 0'880 | 0'4699 | 0'05348 | 5432 |
| 91/13 | 0'092 | 1'012 | 0'6215 | 0'04044 | 7185 |
| 91/12 | 0'104 | 1'144 | 0'7942 | 0'03164 | 9181 |
| 91/11 | 0'116 | 1'276 | 0'9881 | 0'02543 | 11422 |

SQUARES.

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|---|----|----|----|----|----|----|----|----|
| 1'0 | 1'000 | 1'020 | 1'040 | 1'061 | 1'082 | 1'103 | 1'124 | 1'145 | 1'166 | 1'188 | 2 | 4 | 6 | 8 | 10 | 13 | 15 | 17 | 19 |
| 1'1 | 1'210 | 1'232 | 1'254 | 1'277 | 1'300 | 1'323 | 1'346 | 1'369 | 1'392 | 1'416 | 2 | 5 | 7 | 9 | 11 | 14 | 16 | 18 | 21 |
| 1'2 | 1'440 | 1'464 | 1'488 | 1'513 | 1'538 | 1'563 | 1'588 | 1'613 | 1'638 | 1'664 | 2 | 5 | 7 | 10 | 12 | 15 | 17 | 20 | 22 |
| 1'3 | 1'690 | 1'716 | 1'742 | 1'769 | 1'796 | 1'823 | 1'850 | 1'877 | 1'904 | 1'932 | 3 | 5 | 8 | 11 | 13 | 16 | 18 | 22 | 24 |
| 1'4 | 1'960 | 1'988 | 2'016 | 2'045 | 2'074 | 2'103 | 2'132 | 2'161 | 2'190 | 2'220 | 3 | 6 | 9 | 12 | 14 | 17 | 20 | 23 | 26 |
| 1'5 | 2'250 | 2'280 | 2'310 | 2'341 | 2'372 | 2'403 | 2'434 | 2'465 | 2'496 | 2'528 | 3 | 6 | 9 | 12 | 15 | 19 | 22 | 25 | 28 |
| 1'6 | 2'560 | 2'592 | 2'624 | 2'657 | 2'690 | 2'723 | 2'756 | 2'789 | 2'822 | 2'856 | 3 | 7 | 10 | 13 | 16 | 20 | 23 | 26 | 30 |
| 1'7 | 2'890 | 2'924 | 2'958 | 2'993 | 3'028 | 3'063 | 3'098 | 3'133 | 3'168 | 3'204 | 3 | 7 | 10 | 14 | 17 | 21 | 24 | 28 | 31 |
| 1'8 | 3'240 | 3'276 | 3'312 | 3'349 | 3'386 | 3'423 | 3'460 | 3'497 | 3'534 | 3'572 | 4 | 7 | 11 | 15 | 18 | 22 | 26 | 30 | 33 |
| 1'9 | 3'610 | 3'648 | 3'686 | 3'725 | 3'764 | 3'803 | 3'842 | 3'881 | 3'920 | 3'960 | 4 | 8 | 12 | 16 | 19 | 23 | 27 | 31 | 35 |
| 2'0 | 4'000 | 4'040 | 4'080 | 4'121 | 4'162 | 4'203 | 4'244 | 4'285 | 4'326 | 4'368 | 4 | 8 | 12 | 16 | 20 | 25 | 29 | 33 | 37 |
| 2'1 | 4'410 | 4'452 | 4'494 | 4'537 | 4'580 | 4'623 | 4'666 | 4'709 | 4'752 | 3'796 | 4 | 9 | 13 | 17 | 21 | 26 | 30 | 34 | 39 |
| 2'2 | 4'840 | 4'884 | 4'928 | 4'973 | 5'018 | 5'063 | 5'108 | 5'153 | 5'198 | 5'244 | 4 | 9 | 13 | 18 | 22 | 27 | 31 | 36 | 40 |
| 2'3 | 5'290 | 5'336 | 5'382 | 5'429 | 5'476 | 5'523 | 5'570 | 5'617 | 5'664 | 5'712 | 5 | 9 | 14 | 19 | 23 | 28 | 33 | 38 | 42 |
| 2'4 | 5'760 | 5'808 | 5'856 | 5'905 | 5'954 | 6'003 | 6'052 | 6'101 | 6'150 | 6'200 | 5 | 10 | 15 | 20 | 24 | 29 | 34 | 39 | 44 |
| 2'5 | 6'250 | 6'300 | 6'350 | 6'401 | 6'452 | 6'503 | 6'554 | 6'605 | 6'656 | 6'708 | 5 | 10 | 15 | 20 | 25 | 31 | 36 | 41 | 46 |
| 2'6 | 6'760 | 6'812 | 6'864 | 6'917 | 6'970 | 7'023 | 7'076 | 7'129 | 7'182 | 7'236 | 5 | 11 | 16 | 21 | 26 | 32 | 37 | 42 | 48 |
| 2'7 | 7'290 | 7'344 | 7'398 | 7'453 | 7'508 | 7'563 | 7'618 | 7'673 | 7'728 | 7'784 | 5 | 11 | 16 | 22 | 27 | 33 | 38 | 44 | 49 |
| 2'8 | 7'840 | 7'896 | 7'952 | 8'009 | 8'066 | 8'123 | 8'180 | 8'237 | 8'294 | 8'352 | 6 | 11 | 17 | 23 | 28 | 34 | 40 | 46 | 51 |
| 2'9 | 8'410 | 8'468 | 8'526 | 8'585 | 8'644 | 8'703 | 8'762 | 8'821 | 8'880 | 8'940 | 6 | 12 | 18 | 24 | 29 | 35 | 41 | 47 | 53 |
| 3'0 | 9'000 | 9'060 | 9'120 | 9'181 | 9'242 | 9'303 | 9'364 | 9'425 | 9'486 | 9'548 | 6 | 12 | 18 | 24 | 30 | 37 | 43 | 49 | 55 |
| 3'1 | 9'610 | 9'672 | 9'734 | 9'797 | 9'860 | 9'923 | 9'986 | | | | 6 | 13 | 19 | 25 | 31 | 38 | 44 | 50 | 57 |
| 3'1 | | | | | | | | 10'05 | 10'11 | 10'18 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 3'2 | 10'24 | 10'30 | 10'37 | 10'43 | 10'50 | 10'56 | 10'63 | 10'69 | 10'76 | 10'82 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 3'3 | 10'89 | 10'96 | 11'02 | 11'09 | 11'16 | 11'22 | 11'29 | 11'36 | 11'42 | 11'49 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 3'4 | 11'56 | 11'63 | 11'70 | 11'76 | 11'83 | 11'90 | 11'97 | 12'04 | 12'11 | 12'18 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 6 |
| 3'5 | 12'25 | 12'32 | 12'39 | 12'46 | 12'53 | 12'60 | 12'67 | 12'74 | 12'82 | 12'89 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |
| 3'6 | 12'96 | 13'03 | 13'10 | 13'18 | 13'25 | 13'32 | 13'40 | 13'47 | 13'54 | 13'62 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |
| 3'7 | 13'69 | 13'76 | 13'84 | 13'91 | 13'99 | 14'06 | 14'14 | 14'21 | 14'29 | 14'36 | 1 | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |
| 3'8 | 14'44 | 14'52 | 14'59 | 14'67 | 14'75 | 14'82 | 14'90 | 14'98 | 15'05 | 15'13 | 1 | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |
| 3'9 | 15'21 | 15'29 | 15'37 | 15'44 | 15'52 | 15'60 | 15'68 | 15'76 | 15'84 | 15'92 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 6 | 7 |
| 4'0 | 16'00 | 16'08 | 16'16 | 16'24 | 16'32 | 16'40 | 16'48 | 16'56 | 16'65 | 16'73 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 6 | 7 |
| 4'1 | 16'81 | 16'89 | 16'97 | 17'06 | 17'14 | 17'22 | 17'31 | 17'39 | 17'47 | 17'56 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 7 |
| 4'2 | 17'64 | 17'72 | 17'81 | 17'89 | 17'98 | 18'06 | 18'15 | 18'23 | 18'32 | 18'40 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 |
| 4'3 | 18'49 | 18'58 | 18'66 | 18'75 | 18'84 | 18'92 | 19'01 | 19'10 | 19'18 | 19'27 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 |
| 4'4 | 19'36 | 19'45 | 19'54 | 19'62 | 19'71 | 19'80 | 19'89 | 19'98 | 20'07 | 20'16 | 1 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 8 |
| 4'5 | 20'25 | 20'34 | 20'43 | 20'52 | 20'61 | 20'70 | 20'79 | 20'88 | 20'98 | 21'07 | 1 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 8 |
| 4'6 | 21'16 | 21'25 | 21'34 | 21'44 | 21'53 | 21'62 | 21'72 | 21'81 | 21'90 | 22'00 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 7 | 8 |
| 4'7 | 22'09 | 22'18 | 22'28 | 22'37 | 22'47 | 22'56 | 22'66 | 22'75 | 22'85 | 22'94 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 4'8 | 23'04 | 23'14 | 23'23 | 23'33 | 23'43 | 23'52 | 23'62 | 23'72 | 23'81 | 23'91 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 4'9 | 24'01 | 24'11 | 24'21 | 24'30 | 24'40 | 24'50 | 24'60 | 24'70 | 24'80 | 24'90 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 5'0 | 25'00 | 25'10 | 25'20 | 25'30 | 25'40 | 25'50 | 25'60 | 25'70 | 25'81 | 25'91 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 5'1 | 26'01 | 26'11 | 26'21 | 26'32 | 26'42 | 26'52 | 26'63 | 26'73 | 26'83 | 26'94 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 5'2 | 27'04 | 27'14 | 27'25 | 27'35 | 27'46 | 27'56 | 27'67 | 27'77 | 27'88 | 27'98 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 5'3 | 28'09 | 28'20 | 28'30 | 28'41 | 28'52 | 28'62 | 28'73 | 28'84 | 28'94 | 29'05 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 9 | 10 |
| 5'4 | 29'16 | 29'27 | 29'38 | 29'48 | 29'59 | 29'70 | 29'81 | 29'92 | 30'03 | 30'14 | 1 | 2 | 3 | 4 | 6 | 7 | 8 | 9 | 10 |

SQUARES.

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|---|---|---|---|---|---|---|---|----|
| 5'5 | 30'25 | 30'36 | 30'47 | 30'58 | 30'69 | 30'80 | 30'91 | 31'02 | 31'14 | 31'25 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 5'6 | 31'36 | 31'47 | 31'58 | 31'70 | 31'81 | 31'92 | 32'04 | 32'15 | 32'26 | 32'38 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 5'7 | 32'49 | 32'60 | 32'72 | 32'83 | 32'95 | 33'06 | 33'18 | 33'29 | 33'41 | 33'52 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 5'8 | 33'64 | 33'76 | 33'87 | 33'99 | 34'11 | 34'22 | 34'34 | 34'46 | 34'57 | 34'69 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 5'9 | 34'81 | 34'93 | 35'05 | 35'16 | 35'28 | 35'40 | 35'52 | 35'64 | 35'76 | 35'88 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 6'0 | 36'00 | 36'12 | 36'24 | 36'36 | 36'48 | 36'60 | 36'72 | 36'84 | 36'97 | 37'09 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 6'1 | 37'21 | 37'33 | 37'45 | 37'58 | 37'70 | 37'82 | 37'95 | 38'07 | 38'19 | 38'32 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 6'2 | 38'44 | 38'56 | 38'69 | 38'81 | 38'94 | 39'06 | 39'19 | 39'31 | 39'44 | 39'56 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 6'3 | 39'69 | 39'82 | 39'94 | 40'07 | 40'20 | 40'32 | 40'45 | 40'58 | 40'70 | 40'83 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 6'4 | 40'96 | 41'09 | 41'22 | 41'34 | 41'47 | 41'60 | 41'73 | 41'86 | 41'99 | 42'12 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 6'5 | 42'25 | 42'38 | 42'51 | 42'64 | 42'77 | 42'90 | 43'03 | 43'16 | 43'30 | 43'43 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 6'6 | 43'56 | 43'69 | 43'82 | 43'96 | 44'09 | 44'22 | 44'36 | 44'49 | 44'62 | 44'76 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 6'7 | 44'89 | 45'02 | 45'16 | 45'29 | 45'43 | 45'56 | 45'70 | 45'83 | 45'97 | 46'10 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 6'8 | 46'24 | 46'38 | 46'51 | 46'65 | 46'79 | 46'92 | 47'06 | 47'20 | 47'33 | 47'47 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 6'9 | 47'61 | 47'75 | 47'89 | 48'02 | 48'16 | 48'30 | 48'44 | 48'58 | 48'72 | 48'86 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 7'0 | 49'00 | 49'14 | 49'28 | 49'42 | 49'56 | 49'70 | 49'84 | 49'98 | 50'13 | 50'27 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 7'1 | 50'41 | 50'55 | 50'69 | 50'84 | 50'98 | 51'12 | 51'27 | 51'41 | 51'55 | 51'70 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 7'2 | 51'84 | 51'98 | 52'13 | 52'27 | 52'42 | 52'56 | 52'71 | 52'85 | 53'00 | 53'14 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 7'3 | 53'29 | 53'44 | 53'58 | 53'73 | 53'88 | 54'02 | 54'17 | 54'32 | 54'46 | 54'61 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 7'4 | 54'76 | 54'91 | 55'06 | 55'20 | 55'35 | 55'50 | 55'65 | 55'80 | 55'95 | 56'10 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 7'5 | 56'25 | 56'40 | 56'55 | 56'70 | 56'85 | 57'00 | 57'15 | 57'30 | 57'46 | 57'61 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 7'6 | 57'76 | 57'91 | 58'06 | 58'22 | 58'37 | 58'52 | 58'68 | 58'83 | 58'98 | 59'14 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 7'7 | 59'29 | 59'44 | 59'60 | 59'75 | 59'91 | 60'06 | 60'22 | 60'37 | 60'53 | 60'68 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 7'8 | 60'84 | 61'00 | 61'15 | 61'31 | 61'47 | 61'62 | 61'78 | 61'94 | 62'09 | 62'25 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 7'9 | 62'41 | 62'57 | 62'73 | 62'88 | 63'04 | 63'20 | 63'36 | 63'52 | 63'68 | 63'84 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 8'0 | 64'00 | 64'16 | 64'32 | 64'48 | 64'64 | 64'80 | 64'96 | 65'12 | 65'29 | 65'45 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 8'1 | 65'61 | 65'77 | 65'93 | 66'10 | 66'26 | 66'42 | 66'59 | 66'75 | 66'91 | 67'08 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 8'2 | 67'24 | 67'40 | 67'57 | 67'73 | 67'90 | 68'06 | 68'23 | 68'39 | 68'56 | 68'72 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 8'3 | 68'89 | 69'06 | 69'22 | 69'39 | 69'56 | 69'72 | 69'89 | 70'06 | 70'22 | 70'39 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 8'4 | 70'56 | 70'73 | 70'90 | 71'06 | 71'23 | 71'40 | 71'57 | 71'74 | 71'91 | 72'08 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 8'5 | 72'25 | 72'42 | 72'59 | 72'76 | 72'93 | 73'10 | 73'27 | 73'44 | 73'62 | 73'79 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 8'6 | 73'96 | 74'13 | 74'30 | 74'48 | 74'65 | 74'82 | 75'00 | 75'17 | 75'34 | 75'52 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 8'7 | 75'69 | 75'86 | 76'04 | 76'21 | 76'39 | 76'56 | 76'74 | 76'91 | 77'09 | 77'26 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 8'8 | 77'44 | 77'62 | 77'79 | 77'97 | 78'15 | 78'32 | 78'50 | 78'68 | 78'85 | 79'03 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 8'9 | 79'21 | 79'39 | 79'57 | 79'74 | 79'92 | 80'10 | 80'28 | 80'46 | 80'64 | 80'82 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 9'0 | 81'00 | 81'18 | 81'36 | 81'54 | 81'72 | 81'90 | 82'08 | 82'26 | 82'45 | 82'63 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 9'1 | 82'81 | 82'99 | 83'17 | 83'35 | 83'54 | 83'72 | 83'91 | 84'09 | 84'27 | 84'46 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 9'2 | 84'64 | 84'82 | 85'01 | 85'19 | 85'38 | 85'56 | 85'75 | 85'93 | 86'12 | 86'30 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 9'3 | 86'49 | 86'68 | 86'86 | 87'05 | 87'24 | 87'42 | 87'61 | 87'80 | 87'98 | 88'17 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 9'4 | 88'36 | 88'55 | 88'74 | 88'92 | 89'11 | 89'30 | 89'49 | 89'68 | 89'87 | 90'06 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 9'5 | 90'25 | 90'44 | 90'63 | 90'82 | 91'01 | 91'20 | 91'39 | 91'58 | 91'78 | 91'97 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 9'6 | 92'16 | 92'35 | 92'54 | 92'74 | 92'93 | 93'12 | 93'32 | 93'51 | 93'70 | 93'90 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 9'7 | 94'09 | 94'28 | 94'48 | 94'67 | 94'87 | 95'06 | 95'26 | 95'45 | 95'65 | 95'84 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 9'8 | 96'04 | 96'24 | 96'43 | 96'63 | 96'83 | 97'02 | 97'22 | 97'42 | 97'61 | 97'81 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 9'9 | 98'01 | 98'21 | 98'41 | 98'60 | 98'80 | 99'00 | 99'20 | 99'40 | 99'60 | 99'80 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |

RECIPROCAL OF NUMBERS FROM 1000 TO 9999.

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----|-----------|------|------|------|------|------|------|------|------|------|---|----|----|----|----|----|----|----|----|
| 10 | 0.0010000 | 9901 | 9804 | 9709 | 9615 | 9524 | 9434 | 9346 | 9259 | 9174 | 9 | 18 | 27 | 36 | 45 | 55 | 64 | 73 | 82 |
| 11 | 0.0009091 | 9009 | 8929 | 8850 | 8772 | 8696 | 8621 | 8547 | 8475 | 8403 | 8 | 15 | 23 | 30 | 38 | 45 | 53 | 61 | 68 |
| 12 | 0.0008333 | 8264 | 8197 | 8130 | 8065 | 8000 | 7937 | 7874 | 7813 | 7752 | 6 | 13 | 19 | 26 | 32 | 38 | 45 | 51 | 58 |
| 13 | 0.0007692 | 7634 | 7576 | 7519 | 7463 | 7407 | 7353 | 7299 | 7246 | 7194 | 5 | 11 | 16 | 22 | 27 | 33 | 38 | 44 | 49 |
| 14 | 0.0007143 | 7092 | 7042 | 6993 | 6944 | 6897 | 6849 | 6803 | 6757 | 6711 | 5 | 10 | 14 | 19 | 24 | 29 | 33 | 38 | 43 |
| 15 | 0.0006667 | 6623 | 6579 | 6536 | 6494 | 6452 | 6410 | 6369 | 6329 | 6289 | 4 | 8 | 13 | 17 | 21 | 25 | 29 | 33 | 38 |
| 16 | 0.0006250 | 6211 | 6173 | 6135 | 6098 | 6061 | 6024 | 5988 | 5952 | 5917 | 4 | 7 | 11 | 15 | 18 | 22 | 26 | 29 | 33 |
| 17 | 0.0005882 | 5848 | 5814 | 5780 | 5747 | 5714 | 5682 | 5650 | 5618 | 5587 | 3 | 6 | 10 | 13 | 16 | 20 | 23 | 26 | 29 |
| 18 | 0.0005556 | 5525 | 5495 | 5464 | 5435 | 5405 | 5376 | 5348 | 5319 | 5291 | 3 | 6 | 9 | 12 | 15 | 17 | 20 | 23 | 26 |
| 19 | 0.0005263 | 5230 | 5208 | 5181 | 5155 | 5128 | 5102 | 5076 | 5051 | 5025 | 3 | 5 | 8 | 11 | 13 | 16 | 18 | 21 | 24 |
| 20 | 0.0005000 | 4975 | 4950 | 4926 | 4902 | 4878 | 4854 | 4831 | 4808 | 4785 | 2 | 5 | 7 | 10 | 12 | 14 | 17 | 19 | 21 |
| 21 | 0.0004762 | 4739 | 4717 | 4695 | 4673 | 4651 | 4630 | 4608 | 4587 | 4566 | 2 | 4 | 7 | 9 | 11 | 13 | 15 | 17 | 20 |
| 22 | 0.0004545 | 4525 | 4505 | 4484 | 4464 | 4444 | 4425 | 4405 | 4386 | 4367 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| 23 | 0.0004348 | 4329 | 4310 | 4292 | 4274 | 4255 | 4237 | 4219 | 4202 | 4184 | 2 | 4 | 5 | 7 | 9 | 11 | 13 | 14 | 16 |
| 24 | 0.0004167 | 4149 | 4132 | 4115 | 4098 | 4082 | 4065 | 4049 | 4032 | 4016 | 2 | 3 | 5 | 7 | 8 | 10 | 12 | 13 | 15 |
| 25 | 0.0004000 | 3984 | 3968 | 3953 | 3937 | 3922 | 3906 | 3891 | 3876 | 3861 | 2 | 3 | 5 | 6 | 8 | 9 | 11 | 12 | 14 |
| 26 | 0.0003846 | 3831 | 3817 | 3802 | 3788 | 3774 | 3759 | 3745 | 3731 | 3717 | 1 | 3 | 4 | 6 | 7 | 8 | 10 | 11 | 13 |
| 27 | 0.0003704 | 3690 | 3676 | 3663 | 3650 | 3636 | 3623 | 3610 | 3597 | 3584 | 1 | 3 | 4 | 5 | 7 | 8 | 9 | 11 | 12 |
| 28 | 0.0003571 | 3559 | 3546 | 3534 | 3521 | 3509 | 3497 | 3484 | 3472 | 3460 | 1 | 2 | 4 | 5 | 6 | 7 | 9 | 10 | 11 |
| 29 | 0.0003448 | 3436 | 3425 | 3413 | 3401 | 3390 | 3378 | 3367 | 3356 | 3344 | 1 | 2 | 3 | 5 | 6 | 7 | 8 | 9 | 10 |
| 30 | 0.0003333 | 3322 | 3311 | 3300 | 3289 | 3279 | 3268 | 3257 | 3247 | 3236 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 9 | 10 |
| 31 | 0.0003226 | 3215 | 3205 | 3195 | 3185 | 3175 | 3165 | 3155 | 3145 | 3135 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 32 | 0.0003125 | 3115 | 3106 | 3096 | 3086 | 3077 | 3067 | 3058 | 3049 | 3040 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 33 | 0.0003030 | 3021 | 3012 | 3003 | 2994 | 2985 | 2976 | 2967 | 2959 | 2950 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 34 | 0.0002941 | 2933 | 2924 | 2915 | 2907 | 2899 | 2890 | 2882 | 2874 | 2865 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 35 | 0.0002857 | 2849 | 2841 | 2833 | 2825 | 2817 | 2809 | 2801 | 2793 | 2786 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 6 | 7 |
| 36 | 0.0002778 | 2770 | 2762 | 2755 | 2747 | 2740 | 2732 | 2725 | 2717 | 2710 | 1 | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |
| 37 | 0.0002703 | 2695 | 2688 | 2681 | 2674 | 2667 | 2660 | 2653 | 2646 | 2639 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |
| 38 | 0.0002632 | 2625 | 2618 | 2611 | 2604 | 2597 | 2591 | 2584 | 2577 | 2571 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 39 | 0.0002564 | 2558 | 2551 | 2545 | 2538 | 2532 | 2525 | 2519 | 2513 | 2506 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 5 | 6 |
| 40 | 0.0002500 | 2494 | 2488 | 2481 | 2475 | 2469 | 2463 | 2457 | 2451 | 2445 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 41 | 0.0002439 | 2433 | 2427 | 2421 | 2415 | 2410 | 2404 | 2398 | 2392 | 2387 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 5 | 5 |
| 42 | 0.0002381 | 2375 | 2370 | 2364 | 2358 | 2353 | 2347 | 2342 | 2336 | 2331 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 43 | 0.0002326 | 2320 | 2315 | 2309 | 2304 | 2299 | 2294 | 2288 | 2283 | 2278 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 44 | 0.0002273 | 2268 | 2262 | 2257 | 2252 | 2247 | 2242 | 2237 | 2232 | 2227 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 45 | 0.0002222 | 2217 | 2212 | 2208 | 2203 | 2198 | 2193 | 2188 | 2183 | 2179 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 46 | 0.0002174 | 2169 | 2165 | 2160 | 2155 | 2151 | 2146 | 2141 | 2137 | 2132 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 47 | 0.0002128 | 2123 | 2119 | 2114 | 2110 | 2105 | 2101 | 2096 | 2092 | 2088 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 48 | 0.0002083 | 2077 | 2075 | 2070 | 2066 | 2062 | 2058 | 2053 | 2049 | 2045 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 4 |
| 49 | 0.0002041 | 2037 | 2033 | 2028 | 2024 | 2020 | 2016 | 2012 | 2008 | 2004 | 0 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 4 |
| 50 | 0.0002000 | 1996 | 1992 | 1988 | 1984 | 1980 | 1976 | 1972 | 1969 | 1965 | 0 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 4 |
| 51 | 0.0001961 | 1957 | 1953 | 1949 | 1946 | 1942 | 1938 | 1934 | 1931 | 1927 | 0 | 1 | 1 | 2 | 2 | 2 | 3 | 3 | 3 |
| 52 | 0.0001923 | 1919 | 1916 | 1912 | 1908 | 1905 | 1901 | 1898 | 1894 | 1890 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 |
| 53 | 0.0001887 | 1883 | 1880 | 1876 | 1873 | 1869 | 1866 | 1862 | 1859 | 1855 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 |
| 54 | 0.0001852 | 1848 | 1845 | 1842 | 1838 | 1835 | 1832 | 1828 | 1825 | 1821 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 |

N.B.—Three zeros follow the decimal point in the reciprocal of any four figure whole number except the number 1000.

NOTE.—Numbers in difference columns to be subtracted, not added.

RECIPROCAL OF NUMBERS FROM 1000 TO 9999.

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----|-----------|------|------|------|------|------|------|------|------|------|---|---|---|---|---|---|---|---|---|
| 55 | 0'0001818 | 1815 | 1812 | 1808 | 1805 | 1802 | 1799 | 1795 | 1792 | 1789 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 |
| 56 | 0'0001786 | 1783 | 1779 | 1776 | 1773 | 1770 | 1767 | 1764 | 1761 | 1757 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 3 | 3 |
| 57 | 0'0001754 | 1751 | 1748 | 1745 | 1742 | 1739 | 1736 | 1733 | 1730 | 1727 | 0 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |
| 58 | 0'0001724 | 1721 | 1718 | 1715 | 1712 | 1709 | 1706 | 1704 | 1701 | 1698 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 3 |
| 59 | 0'0001695 | 1692 | 1689 | 1686 | 1684 | 1681 | 1678 | 1675 | 1672 | 1669 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 3 |
| 60 | 0'0001667 | 1664 | 1661 | 1658 | 1656 | 1653 | 1650 | 1647 | 1645 | 1642 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 3 |
| 61 | 0'0001639 | 1637 | 1634 | 1631 | 1629 | 1626 | 1623 | 1621 | 1618 | 1616 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| 62 | 0'0001613 | 1610 | 1608 | 1605 | 1603 | 1600 | 1597 | 1595 | 1592 | 1590 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| 63 | 0'0001587 | 1585 | 1582 | 1580 | 1577 | 1575 | 1572 | 1570 | 1567 | 1565 | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| 64 | 0'0001563 | 1560 | 1558 | 1555 | 1553 | 1550 | 1548 | 1546 | 1543 | 1541 | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| 65 | 0'0001538 | 1536 | 1534 | 1531 | 1529 | 1527 | 1524 | 1522 | 1520 | 1517 | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| 66 | 0'0001515 | 1513 | 1511 | 1508 | 1506 | 1504 | 1502 | 1499 | 1497 | 1495 | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| 67 | 0'0001493 | 1490 | 1488 | 1486 | 1484 | 1481 | 1479 | 1477 | 1475 | 1473 | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| 68 | 0'0001471 | 1468 | 1466 | 1464 | 1462 | 1460 | 1458 | 1456 | 1453 | 1451 | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| 69 | 0'0001449 | 1447 | 1445 | 1443 | 1441 | 1439 | 1437 | 1435 | 1433 | 1431 | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| 70 | 0'0001429 | 1427 | 1425 | 1422 | 1420 | 1418 | 1416 | 1414 | 1412 | 1410 | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| 71 | 0'0001408 | 1406 | 1404 | 1403 | 1401 | 1399 | 1397 | 1395 | 1393 | 1391 | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| 72 | 0'0001389 | 1387 | 1385 | 1383 | 1381 | 1379 | 1377 | 1376 | 1374 | 1372 | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| 73 | 0'0001370 | 1368 | 1366 | 1364 | 1362 | 1361 | 1359 | 1357 | 1355 | 1353 | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| 74 | 0'0001351 | 1350 | 1348 | 1346 | 1344 | 1342 | 1340 | 1339 | 1337 | 1335 | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| 75 | 0'0001333 | 1332 | 1330 | 1328 | 1326 | 1325 | 1323 | 1321 | 1319 | 1318 | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| 76 | 0'0001316 | 1314 | 1312 | 1311 | 1309 | 1307 | 1305 | 1304 | 1302 | 1300 | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| 77 | 0'0001299 | 1297 | 1295 | 1294 | 1292 | 1290 | 1289 | 1287 | 1285 | 1284 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 |
| 78 | 0'0001282 | 1280 | 1279 | 1277 | 1276 | 1274 | 1272 | 1271 | 1269 | 1267 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 |
| 79 | 0'0001266 | 1264 | 1263 | 1261 | 1259 | 1258 | 1256 | 1255 | 1253 | 1252 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 |
| 80 | 0'0001250 | 1248 | 1247 | 1245 | 1244 | 1242 | 1241 | 1239 | 1238 | 1236 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 |
| 81 | 0'0001235 | 1233 | 1232 | 1230 | 1229 | 1227 | 1225 | 1224 | 1222 | 1221 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 |
| 82 | 0'0001220 | 1218 | 1217 | 1215 | 1214 | 1212 | 1211 | 1209 | 1208 | 1206 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 |
| 83 | 0'0001205 | 1203 | 1202 | 1200 | 1199 | 1198 | 1196 | 1195 | 1193 | 1192 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 |
| 84 | 0'0001190 | 1189 | 1188 | 1186 | 1185 | 1183 | 1182 | 1181 | 1179 | 1178 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 |
| 85 | 0'0001176 | 1175 | 1174 | 1172 | 1171 | 1170 | 1168 | 1167 | 1166 | 1164 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 |
| 86 | 0'0001163 | 1161 | 1160 | 1159 | 1157 | 1156 | 1155 | 1153 | 1152 | 1151 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 |
| 87 | 0'0001149 | 1148 | 1147 | 1145 | 1144 | 1143 | 1142 | 1140 | 1139 | 1138 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 |
| 88 | 0'0001136 | 1135 | 1134 | 1133 | 1131 | 1130 | 1129 | 1127 | 1126 | 1125 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 |
| 89 | 0'0001124 | 1122 | 1121 | 1120 | 1119 | 1117 | 1116 | 1115 | 1114 | 1112 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 |
| 90 | 0'0001111 | 1110 | 1109 | 1107 | 1106 | 1105 | 1104 | 1103 | 1101 | 1100 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 2 | 2 |
| 91 | 0'0001099 | 1098 | 1096 | 1095 | 1094 | 1093 | 1092 | 1091 | 1089 | 1088 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 2 | 2 |
| 92 | 0'0001087 | 1086 | 1085 | 1083 | 1082 | 1081 | 1080 | 1079 | 1078 | 1076 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 2 | 2 |
| 93 | 0'0001075 | 1074 | 1073 | 1072 | 1071 | 1070 | 1068 | 1067 | 1066 | 1065 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 2 | 2 |
| 94 | 0'0001064 | 1063 | 1062 | 1060 | 1059 | 1058 | 1057 | 1056 | 1055 | 1054 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 2 | 2 |
| 95 | 0'0001053 | 1052 | 1050 | 1049 | 1048 | 1047 | 1046 | 1045 | 1044 | 1043 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 2 | 2 |
| 96 | 0'0001042 | 1041 | 1040 | 1038 | 1037 | 1036 | 1035 | 1034 | 1033 | 1032 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 2 | 2 |
| 97 | 0'0001031 | 1030 | 1029 | 1028 | 1027 | 1026 | 1025 | 1024 | 1022 | 1021 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 2 | 2 |
| 98 | 0'0001020 | 1019 | 1018 | 1017 | 1016 | 1015 | 1014 | 1013 | 1012 | 1011 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 2 | 2 |
| 99 | 0'0001010 | 1009 | 1008 | 1007 | 1006 | 1005 | 1004 | 1003 | 1002 | 1001 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 2 | 2 |

N.B.—Three zeros follow the decimal point in the reciprocal of any four-figure whole number except the number 1000.

NOTE.—Numbers in difference columns to be subtracted, not added.

NATURAL TANGENTS.

| | °0 | °1 | °2 | °3 | °4 | °5 | °6 | °7 | °8 | °9 |
|----|-------|------|------|------|------|------|------|------|------|------|
| 0° | ·0000 | 0017 | 0035 | 0052 | 0070 | 0087 | 0105 | 0122 | 0140 | 0157 |
| 1 | ·0175 | 0192 | 0209 | 0227 | 0244 | 0262 | 0·79 | 0297 | 0314 | 0332 |
| 2 | ·0349 | 0367 | 0384 | 0402 | 0419 | 0437 | 0454 | 0472 | 0489 | 0507 |
| 3 | ·0524 | 0542 | 0559 | 0577 | 0594 | 0612 | 0629 | 0647 | 0664 | 0682 |
| 4 | ·0699 | 0717 | 0734 | 0752 | 0769 | 0787 | 0805 | 0822 | 0840 | 0857 |
| 5 | ·0875 | 0892 | 0910 | 0928 | 0945 | 0963 | 0981 | 0998 | 1016 | 1033 |
| 6 | ·1051 | 1069 | 1086 | 1104 | 1122 | 1139 | 1157 | 1175 | 1192 | 1210 |
| 7 | ·1228 | 1246 | 1263 | 1281 | 1299 | 1317 | 1334 | 1352 | 1370 | 1388 |
| 8 | ·1405 | 1423 | 1441 | 1459 | 1477 | 1495 | 1512 | 1530 | 1548 | 1566 |
| 9 | ·1584 | 1602 | 1620 | 1638 | 1655 | 1673 | 1691 | 1709 | 1727 | 1745 |
| 10 | ·1763 | 1781 | 1799 | 1817 | 1835 | 1853 | 1871 | 1890 | 1908 | 1926 |
| 11 | ·1944 | 1962 | 1980 | 1998 | 2016 | 2035 | 2053 | 2071 | 2089 | 2107 |
| 12 | ·2126 | 2144 | 2162 | 2180 | 2199 | 2217 | 2235 | 2254 | 2272 | 2290 |
| 13 | ·2309 | 2327 | 2345 | 2364 | 2382 | 2401 | 2419 | 2438 | 2456 | 2475 |
| 14 | ·2493 | 2512 | 2530 | 2549 | 2568 | 2586 | 2605 | 2623 | 2642 | 2661 |
| 15 | ·2679 | 2698 | 2717 | 2736 | 2754 | 2773 | 2792 | 2811 | 2830 | 2849 |
| 16 | ·2867 | 2886 | 2905 | 2924 | 2943 | 2962 | 2981 | 3000 | 3019 | 3038 |
| 17 | ·3057 | 3076 | 3096 | 3115 | 3134 | 3153 | 3172 | 3191 | 3211 | 3230 |
| 18 | ·3249 | 3269 | 3288 | 3307 | 3327 | 3346 | 3365 | 3385 | 3404 | 3424 |
| 19 | ·3443 | 3463 | 3482 | 3502 | 3522 | 3541 | 3561 | 3581 | 3600 | 3620 |
| 20 | ·3640 | 3659 | 3679 | 3699 | 3719 | 3739 | 3759 | 3779 | 3799 | 3819 |
| 21 | ·3839 | 3859 | 3879 | 3899 | 3919 | 3939 | 3959 | 3979 | 4000 | 4020 |
| 22 | ·4040 | 4061 | 4081 | 4101 | 4122 | 4142 | 4163 | 4183 | 4204 | 4224 |
| 23 | ·4245 | 4265 | 4286 | 4307 | 4327 | 4348 | 4369 | 4390 | 4411 | 4431 |
| 24 | ·4452 | 4473 | 4494 | 4515 | 4536 | 4557 | 4578 | 4599 | 4621 | 4642 |
| 25 | ·4663 | 4684 | 4706 | 4727 | 4748 | 4770 | 4791 | 4813 | 4834 | 4856 |
| 26 | ·4877 | 4899 | 4921 | 4942 | 4964 | 4986 | 5008 | 5029 | 5051 | 5073 |
| 27 | ·5095 | 5117 | 5139 | 5161 | 5184 | 5206 | 5228 | 5250 | 5272 | 5295 |
| 28 | ·5317 | 5340 | 5362 | 5384 | 5407 | 5430 | 5452 | 5475 | 5498 | 5520 |
| 29 | ·5543 | 5566 | 5589 | 5612 | 5635 | 5658 | 5681 | 5704 | 5727 | 5750 |
| 30 | ·5774 | 5797 | 5820 | 5844 | 5867 | 5890 | 5914 | 5938 | 5961 | 5985 |
| 31 | ·6009 | 6032 | 6056 | 6080 | 6104 | 6128 | 6152 | 6176 | 6200 | 6224 |
| 32 | ·6249 | 6273 | 6297 | 6322 | 6346 | 6371 | 6395 | 6420 | 6445 | 6469 |
| 33 | ·6494 | 6519 | 6544 | 6569 | 6594 | 6619 | 6644 | 6669 | 6694 | 6720 |
| 34 | ·6745 | 6771 | 6796 | 6822 | 6847 | 6873 | 6899 | 6924 | 6950 | 6976 |
| 35 | ·7002 | 7028 | 7054 | 7080 | 7107 | 7133 | 7159 | 7186 | 7212 | 7239 |
| 36 | ·7265 | 7292 | 7319 | 7346 | 7373 | 7400 | 7427 | 7454 | 7481 | 7508 |
| 37 | ·7536 | 7563 | 7590 | 7618 | 7646 | 7673 | 7701 | 7729 | 7757 | 7785 |
| 38 | ·7813 | 7841 | 7869 | 7898 | 7926 | 7954 | 7983 | 8012 | 8040 | 8069 |
| 39 | ·8098 | 8127 | 8156 | 8185 | 8214 | 8243 | 8273 | 8302 | 8332 | 8361 |
| 40 | ·8391 | 8421 | 8451 | 8481 | 8511 | 8541 | 8571 | 8601 | 8632 | 8662 |
| 41 | ·8693 | 8724 | 8754 | 8785 | 8816 | 8847 | 8878 | 8910 | 8941 | 8972 |
| 42 | ·9004 | 9036 | 9067 | 9099 | 9131 | 9163 | 9195 | 9228 | 9260 | 9293 |
| 43 | ·9325 | 9358 | 9391 | 9424 | 9457 | 9490 | 9523 | 9556 | 9590 | 9623 |
| 44 | ·9657 | 9691 | 9725 | 9759 | 9793 | 9827 | 9861 | 9896 | 9930 | 9965 |

NATURAL TANGENTS.

| | °0 | °1 | °2 | °3 | °4 | °5 | °6 | °7 | °8 | °9 |
|-----|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 45° | 1°0000 | 0035 | 0070 | 0105 | 0141 | 0176 | 0212 | 0247 | 0283 | 0319 |
| 46 | 1°0355 | 0392 | 0428 | 0464 | 0501 | 0538 | 0575 | 0612 | 0649 | 0686 |
| 47 | 1°0724 | 0761 | 0799 | 0837 | 0875 | 0913 | 0951 | 0990 | 1028 | 1067 |
| 48 | 1°1106 | 1145 | 1184 | 1224 | 1263 | 1303 | 1343 | 1383 | 1423 | 1463 |
| 49 | 1°1504 | 1544 | 1585 | 1626 | 1667 | 1708 | 1750 | 1792 | 1833 | 1875 |
| 50 | 1°1918 | 1960 | 2002 | 2045 | 2088 | 2131 | 2174 | 2218 | 2261 | 2305 |
| 51 | 1°2349 | 2393 | 2437 | 2482 | 2527 | 2572 | 2617 | 2662 | 2708 | 2753 |
| 52 | 1°2799 | 2846 | 2892 | 2938 | 2985 | 3032 | 3079 | 3127 | 3175 | 3222 |
| 53 | 1°3270 | 3319 | 3367 | 3416 | 3465 | 3514 | 3564 | 3613 | 3663 | 3713 |
| 54 | 1°3764 | 3814 | 3865 | 3916 | 3968 | 4019 | 4071 | 4124 | 4176 | 4229 |
| 55 | 1°4281 | 4335 | 4388 | 4442 | 4496 | 4550 | 4605 | 4659 | 4715 | 4770 |
| 56 | 1°4826 | 4882 | 4938 | 4994 | 5051 | 5108 | 5166 | 5224 | 5282 | 5340 |
| 57 | 1°5399 | 5458 | 5517 | 5577 | 5637 | 5697 | 5757 | 5818 | 5880 | 5941 |
| 58 | 1°6003 | 6066 | 6128 | 6191 | 6255 | 6319 | 6383 | 6447 | 6512 | 6577 |
| 59 | 1°6643 | 6709 | 6775 | 6842 | 6909 | 6977 | 7045 | 7113 | 7182 | 7251 |
| 60 | 1°7321 | 7391 | 7461 | 7532 | 7603 | 7675 | 7747 | 7820 | 7893 | 7966 |
| 61 | 1°8040 | 8115 | 8190 | 8265 | 8341 | 8418 | 8495 | 8572 | 8650 | 8728 |
| 62 | 1°8807 | 8887 | 8907 | 9047 | 9128 | 9210 | 9292 | 9375 | 9458 | 9542 |
| 63 | 1°9626 | 9711 | 9797 | 9883 | 9970 | 0057 | 0145 | 0233 | 0323 | 0413 |
| 64 | 2°0503 | 0594 | 0686 | 0778 | 0872 | 0965 | 1060 | 1155 | 1251 | 1348 |
| 65 | 2°1445 | 1543 | 1642 | 1742 | 1842 | 1943 | 2045 | 2148 | 2251 | 2355 |
| 66 | 2°2460 | 2566 | 2673 | 2781 | 2889 | 2998 | 3109 | 3220 | 3332 | 3445 |
| 67 | 2°3559 | 3673 | 3789 | 3906 | 4023 | 4142 | 4262 | 4383 | 4504 | 4627 |
| 68 | 2°4751 | 4876 | 5002 | 5129 | 5257 | 5386 | 5517 | 5649 | 5782 | 5916 |
| 69 | 2°6051 | 6187 | 6325 | 6464 | 6605 | 6746 | 6889 | 7034 | 7179 | 7326 |
| 70 | 2°7475 | 7625 | 7776 | 7929 | 8083 | 8239 | 8397 | 8556 | 8716 | 8878 |
| 71 | 2°9042 | 9208 | 9375 | 9544 | 9714 | 9887 | 0061 | 0237 | 0415 | 0595 |
| 72 | 3°0777 | 0961 | 1146 | 1334 | 1524 | 1716 | 1910 | 2106 | 2305 | 2506 |
| 73 | 3°2709 | 2914 | 3122 | 3332 | 3544 | 3759 | 3977 | 4197 | 4420 | 4646 |
| 74 | 3°4874 | 5105 | 5339 | 5576 | 5816 | 6059 | 6305 | 6554 | 6806 | 7062 |
| 75 | 3°7321 | 7583 | 7848 | 8118 | 8391 | 8667 | 8947 | 9232 | 9520 | 9812 |
| 76 | 4°0108 | 0408 | 0713 | 1022 | 1335 | 1653 | 1976 | 2303 | 2635 | 2972 |
| 77 | 4°3315 | 3662 | 4015 | 4374 | 4737 | 5107 | 5483 | 5864 | 6252 | 6646 |
| 78 | 4°7046 | 7453 | 7867 | 8288 | 8716 | 9152 | 9594 | 0045 | 0504 | 0970 |
| 79 | 5°1446 | 1929 | 2422 | 2924 | 3435 | 3955 | 4486 | 5026 | 5576 | 6140 |
| 80 | 5°6713 | 7297 | 7894 | 8502 | 9124 | 9758 | 0405 | 1066 | 1742 | 2432 |
| 81 | 6°3138 | 3859 | 4596 | 5350 | 6122 | 6912 | 7720 | 8548 | 9395 | 0264 |
| 82 | 7°1154 | 2066 | 3002 | 3952 | 4947 | 5958 | 6996 | 8062 | 9158 | 0285 |
| 83 | 8°1443 | 2636 | 3863 | 5126 | 6427 | 7769 | 9152 | 0579 | 2052 | 3572 |
| 84 | 9°5144 | 9°677 | 9°845 | 10°02 | 10°20 | 10°39 | 10°58 | 10°78 | 10°99 | 11°20 |
| 85 | 11°43 | 11°66 | 11°91 | 12°16 | 12°43 | 12°71 | 13°00 | 13°30 | 13°62 | 13°95 |
| 86 | 14°30 | 14°67 | 15°06 | 15°46 | 15°89 | 16°35 | 16°83 | 17°34 | 17°89 | 18°46 |
| 87 | 19°08 | 19°74 | 20°45 | 21°20 | 22°02 | 22°90 | 23°86 | 24°90 | 26°03 | 27°27 |
| 88 | 28°64 | 30°14 | 31°82 | 33°69 | 35°80 | 38°19 | 40°92 | 44°07 | 47°74 | 52°08 |
| 89 | 57°29 | 63°66 | 71°62 | 81°85 | 95°49 | 114°6 | 143°2 | 191°0 | 286°5 | 573°0 |

NATURAL SINES.

| | °0 | °1 | °2 | °3 | °4 | °5 | °6 | °7 | °8 | °9 |
|----|------|------|------|------|------|------|------|------|------|------|
| 0° | 0000 | 0017 | 0035 | 0052 | 0070 | 0087 | 0105 | 0122 | 0140 | 0157 |
| 1 | 0175 | 0192 | 0209 | 0227 | 0244 | 0262 | 0279 | 0297 | 0314 | 0332 |
| 2 | 0349 | 0366 | 0384 | 0401 | 0419 | 0436 | 0454 | 0471 | 0488 | 0506 |
| 3 | 0523 | 0541 | 0558 | 0576 | 0593 | 0610 | 0628 | 0645 | 0663 | 0680 |
| 4 | 0698 | 0715 | 0732 | 0750 | 0767 | 0785 | 0802 | 0819 | 0837 | 0854 |
| 5 | 0872 | 0889 | 0906 | 0924 | 0941 | 0958 | 0976 | 0993 | 1011 | 1028 |
| 6 | 1045 | 1063 | 1080 | 1097 | 1115 | 1132 | 1149 | 1167 | 1184 | 1201 |
| 7 | 1219 | 1236 | 1253 | 1271 | 1288 | 1305 | 1323 | 1340 | 1357 | 1374 |
| 8 | 1392 | 1409 | 1426 | 1444 | 1461 | 1478 | 1495 | 1513 | 1530 | 1547 |
| 9 | 1564 | 1582 | 1599 | 1616 | 1633 | 1650 | 1668 | 1685 | 1702 | 1719 |
| 10 | 1736 | 1754 | 1771 | 1788 | 1805 | 1822 | 1840 | 1857 | 1874 | 1891 |
| 11 | 1908 | 1925 | 1942 | 1959 | 1977 | 1994 | 2011 | 2028 | 2045 | 2062 |
| 12 | 2079 | 2096 | 2113 | 2130 | 2147 | 2164 | 2181 | 2198 | 2215 | 2232 |
| 13 | 2250 | 2267 | 2284 | 2300 | 2317 | 2334 | 2351 | 2368 | 2385 | 2402 |
| 14 | 2419 | 2436 | 2453 | 2470 | 2487 | 2504 | 2521 | 2538 | 2554 | 2571 |
| 15 | 2588 | 2605 | 2622 | 2639 | 2656 | 2672 | 2689 | 2706 | 2723 | 2740 |
| 16 | 2756 | 2773 | 2790 | 2807 | 2823 | 2840 | 2857 | 2874 | 2890 | 2907 |
| 17 | 2924 | 2940 | 2957 | 2974 | 2990 | 3007 | 3024 | 3040 | 3057 | 3074 |
| 18 | 3090 | 3107 | 3123 | 3140 | 3156 | 3173 | 3190 | 3206 | 3223 | 3239 |
| 19 | 3256 | 3272 | 3289 | 3305 | 3322 | 3338 | 3355 | 3371 | 3387 | 3404 |
| 20 | 3420 | 3437 | 3453 | 3469 | 3486 | 3502 | 3518 | 3535 | 3551 | 3567 |
| 21 | 3584 | 3600 | 3616 | 3633 | 3649 | 3665 | 3681 | 3697 | 3714 | 3730 |
| 22 | 3746 | 3762 | 3778 | 3795 | 3811 | 3827 | 3843 | 3859 | 3875 | 3891 |
| 23 | 3907 | 3923 | 3939 | 3955 | 3971 | 3987 | 4003 | 4019 | 4035 | 4051 |
| 24 | 4067 | 4083 | 4099 | 4115 | 4131 | 4147 | 4163 | 4179 | 4195 | 4210 |
| 25 | 4226 | 4242 | 4258 | 4274 | 4289 | 4305 | 4321 | 4337 | 4352 | 4368 |
| 26 | 4384 | 4399 | 4415 | 4431 | 4446 | 4462 | 4478 | 4493 | 4509 | 4524 |
| 27 | 4540 | 4555 | 4571 | 4586 | 4602 | 4617 | 4633 | 4648 | 4664 | 4679 |
| 28 | 4695 | 4710 | 4726 | 4741 | 4756 | 4772 | 4787 | 4802 | 4818 | 4833 |
| 29 | 4848 | 4863 | 4879 | 4894 | 4909 | 4924 | 4939 | 4955 | 4970 | 4985 |
| 30 | 5000 | 5015 | 5030 | 5045 | 5060 | 5075 | 5090 | 5105 | 5120 | 5135 |
| 31 | 5150 | 5165 | 5180 | 5195 | 5210 | 5225 | 5240 | 5255 | 5270 | 5284 |
| 32 | 5299 | 5314 | 5329 | 5344 | 5358 | 5373 | 5388 | 5402 | 5417 | 5432 |
| 33 | 5446 | 5461 | 5476 | 5490 | 5505 | 5519 | 5534 | 5548 | 5563 | 5577 |
| 34 | 5592 | 5606 | 5621 | 5635 | 5650 | 5664 | 5678 | 5693 | 5707 | 5721 |
| 35 | 5736 | 5750 | 5764 | 5779 | 5793 | 5807 | 5821 | 5835 | 5850 | 5864 |
| 36 | 5878 | 5892 | 5906 | 5920 | 5934 | 5948 | 5962 | 5976 | 5990 | 6004 |
| 37 | 6018 | 6032 | 6046 | 6060 | 6074 | 6088 | 6101 | 6115 | 6129 | 6143 |
| 38 | 6157 | 6170 | 6184 | 6198 | 6211 | 6225 | 6239 | 6252 | 6266 | 6280 |
| 39 | 6293 | 6307 | 6320 | 6334 | 6347 | 6361 | 6374 | 6388 | 6401 | 6414 |
| 40 | 6428 | 6441 | 6455 | 6468 | 6481 | 6494 | 6508 | 6521 | 6534 | 6547 |
| 41 | 6561 | 6574 | 6587 | 6600 | 6613 | 6626 | 6639 | 6652 | 6665 | 6678 |
| 42 | 6691 | 6704 | 6717 | 6730 | 6743 | 6756 | 6769 | 6782 | 6794 | 6807 |
| 43 | 6820 | 6833 | 6845 | 6858 | 6871 | 6884 | 6896 | 6909 | 6921 | 6934 |
| 44 | 6947 | 6959 | 6972 | 6984 | 6997 | 7009 | 7022 | 7034 | 7046 | 7059 |

NATURAL SINES.

| | °0 | °1 | °2 | °3 | °4 | °5 | °6 | °7 | °8 | °9 |
|----|------|------|------|------|------|---------|---------|---------|---------|---------|
| 45 | 7071 | 7083 | 7096 | 7108 | 7120 | 7133 | 7145 | 7157 | 7169 | 7181 |
| 46 | 7193 | 7206 | 7218 | 7230 | 7242 | 7254 | 7266 | 7278 | 7290 | 7302 |
| 47 | 7314 | 7325 | 7337 | 7349 | 7361 | 7373 | 7385 | 7396 | 7408 | 7420 |
| 48 | 7431 | 7443 | 7455 | 7466 | 7478 | 7490 | 7501 | 7513 | 7524 | 7536 |
| 49 | 7547 | 7558 | 7570 | 7581 | 7593 | 7604 | 7615 | 7627 | 7638 | 7649 |
| 50 | 7660 | 7672 | 7683 | 7694 | 7705 | 7716 | 7727 | 7738 | 7749 | 7760 |
| 51 | 7771 | 7782 | 7793 | 7804 | 7815 | 7826 | 7837 | 7848 | 7859 | 7869 |
| 52 | 7880 | 7891 | 7902 | 7912 | 7923 | 7934 | 7944 | 7955 | 7965 | 7976 |
| 53 | 7986 | 7997 | 8007 | 8018 | 8028 | 8039 | 8049 | 8059 | 8070 | 8080 |
| 54 | 8090 | 8100 | 8111 | 8121 | 8131 | 8141 | 8151 | 8161 | 8171 | 8181 |
| 55 | 8192 | 8202 | 8211 | 8221 | 8231 | 8241 | 8251 | 8261 | 8271 | 8281 |
| 56 | 8290 | 8300 | 8310 | 8320 | 8329 | 8339 | 8348 | 8358 | 8368 | 8377 |
| 57 | 8387 | 8396 | 8406 | 8415 | 8425 | 8434 | 8443 | 8453 | 8462 | 8471 |
| 58 | 8480 | 8490 | 8499 | 8508 | 8517 | 8526 | 8536 | 8545 | 8554 | 8563 |
| 59 | 8572 | 8581 | 8590 | 8599 | 8607 | 8616 | 8625 | 8634 | 8643 | 8652 |
| 60 | 8660 | 8669 | 8678 | 8686 | 8695 | 8704 | 8712 | 8721 | 8729 | 8738 |
| 61 | 8746 | 8755 | 8763 | 8771 | 8780 | 8788 | 8796 | 8805 | 8813 | 8821 |
| 62 | 8829 | 8838 | 8846 | 8854 | 8862 | 8870 | 8878 | 8886 | 8894 | 8902 |
| 63 | 8910 | 8918 | 8926 | 8934 | 8942 | 8949 | 8957 | 8965 | 8973 | 8980 |
| 64 | 8988 | 8996 | 9003 | 9011 | 9018 | 9026 | 9033 | 9041 | 9048 | 9056 |
| 65 | 9063 | 9070 | 9078 | 9085 | 9092 | 9100 | 9107 | 9114 | 9121 | 9128 |
| 66 | 9135 | 9143 | 9150 | 9157 | 9164 | 9171 | 9178 | 9184 | 9191 | 9198 |
| 67 | 9205 | 9212 | 9219 | 9225 | 9232 | 9239 | 9245 | 9252 | 9259 | 9265 |
| 68 | 9272 | 9278 | 9285 | 9291 | 9298 | 9304 | 9311 | 9317 | 9323 | 9330 |
| 69 | 9336 | 9342 | 9348 | 9354 | 9361 | 9367 | 9373 | 9379 | 9385 | 9391 |
| 70 | 9397 | 9403 | 9409 | 9415 | 9421 | 9426 | 9432 | 9438 | 9444 | 9449 |
| 71 | 9455 | 9461 | 9466 | 9472 | 9478 | 9483 | 9489 | 9494 | 9500 | 9505 |
| 72 | 9511 | 9516 | 9521 | 9527 | 9532 | 9537 | 9542 | 9548 | 9553 | 9558 |
| 73 | 9563 | 9568 | 9573 | 9578 | 9583 | 9588 | 9593 | 9598 | 9603 | 9608 |
| 74 | 9613 | 9617 | 9622 | 9627 | 9632 | 9636 | 9641 | 9646 | 9650 | 9655 |
| 75 | 9659 | 9664 | 9668 | 9673 | 9677 | 9681 | 9686 | 9690 | 9694 | 9699 |
| 76 | 9703 | 9707 | 9711 | 9715 | 9720 | 9724 | 9728 | 9732 | 9736 | 9740 |
| 77 | 9744 | 9748 | 9751 | 9755 | 9759 | 9763 | 9767 | 9770 | 9774 | 9778 |
| 78 | 9781 | 9785 | 9789 | 9792 | 9796 | 9799 | 9803 | 9806 | 9810 | 9813 |
| 79 | 9816 | 9820 | 9823 | 9826 | 9829 | 9833 | 9836 | 9839 | 9842 | 9845 |
| 80 | 9848 | 9851 | 9854 | 9857 | 9860 | 9863 | 9866 | 9869 | 9871 | 9874 |
| 81 | 9877 | 9880 | 9882 | 9885 | 9888 | 9890 | 9893 | 9895 | 9898 | 9900 |
| 82 | 9903 | 9905 | 9907 | 9910 | 9912 | 9914 | 9917 | 9919 | 9921 | 9923 |
| 83 | 9925 | 9928 | 9930 | 9932 | 9934 | 9936 | 9938 | 9940 | 9942 | 9943 |
| 84 | 9945 | 9947 | 9949 | 9951 | 9952 | 9954 | 9956 | 9957 | 9959 | 9960 |
| 85 | 9962 | 9963 | 9965 | 9966 | 9968 | 9969 | 9971 | 9972 | 9973 | 9974 |
| 86 | 9976 | 9977 | 9978 | 9979 | 9980 | 9981 | 9982 | 9983 | 9984 | 9985 |
| 87 | 9986 | 9987 | 9988 | 9989 | 9990 | 9990 | 9991 | 9992 | 9993 | 9993 |
| 88 | 9994 | 9995 | 9995 | 9996 | 9996 | 9997 | 9997 | 9997 | 9998 | 9998 |
| 89 | 9998 | 9999 | 9999 | 9999 | 9999 | 1'000 | 1'000 | 1'000 | 1'000 | 1'000 |
| | | | | | | nearly. | nearly. | nearly. | nearly. | nearly. |

LOGARITHMS.

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 12 | 3 4 | 5 | 6 7 | 8 9 |
|----|------|------|------|------|------|------|------|------|------|------|-----|-------|----|-------|-------|
| 10 | 0000 | 0043 | 0086 | 0128 | 0171 | 0212 | 0253 | 0294 | 0334 | 0374 | 4 8 | 12 17 | 21 | 25 29 | 33 37 |
| 11 | 0414 | 0453 | 0492 | 0531 | 0569 | 0607 | 0645 | 0682 | 0719 | 0755 | 4 8 | 11 15 | 19 | 23 26 | 30 34 |
| 12 | 0792 | 0828 | 0864 | 0899 | 0934 | 0969 | 1004 | 1038 | 1072 | 1106 | 3 7 | 10 14 | 17 | 21 24 | 28 31 |
| 13 | 1139 | 1173 | 1206 | 1239 | 1271 | 1303 | 1335 | 1367 | 1399 | 1430 | 3 6 | 10 13 | 16 | 19 23 | 26 29 |
| 14 | 1461 | 1492 | 1523 | 1553 | 1584 | 1614 | 1644 | 1673 | 1703 | 1732 | 3 6 | 9 12 | 15 | 18 21 | 24 27 |
| 15 | 1761 | 1790 | 1818 | 1847 | 1875 | 1903 | 1931 | 1959 | 1987 | 2014 | 3 6 | 8 11 | 14 | 17 20 | 22 25 |
| 16 | 2041 | 2068 | 2095 | 2122 | 2148 | 2175 | 2201 | 2227 | 2253 | 2279 | 3 5 | 8 11 | 13 | 16 18 | 21 24 |
| 17 | 2304 | 2330 | 2355 | 2380 | 2403 | 2430 | 2455 | 2480 | 2504 | 2529 | 2 5 | 7 10 | 12 | 15 17 | 20 22 |
| 18 | 2553 | 2577 | 2601 | 2625 | 2648 | 2672 | 2695 | 2718 | 2742 | 2765 | 2 5 | 7 9 | 12 | 14 16 | 19 21 |
| 19 | 2788 | 2810 | 2833 | 2856 | 2878 | 2900 | 2923 | 2945 | 2967 | 2989 | 2 4 | 7 9 | 11 | 13 16 | 18 20 |
| 20 | 3010 | 3032 | 3054 | 3075 | 3096 | 3118 | 3139 | 3160 | 3181 | 3201 | 2 4 | 6 8 | 11 | 13 15 | 17 19 |
| 21 | 3222 | 3243 | 3263 | 3284 | 3304 | 3324 | 3345 | 3365 | 3385 | 3404 | 2 4 | 6 8 | 10 | 12 14 | 16 18 |
| 22 | 3424 | 3444 | 3464 | 3483 | 3502 | 3522 | 3541 | 3560 | 3579 | 3598 | 2 4 | 6 8 | 10 | 12 14 | 15 17 |
| 23 | 3617 | 3636 | 3655 | 3674 | 3692 | 3711 | 3729 | 3747 | 3766 | 3784 | 2 4 | 6 7 | 9 | 11 13 | 15 17 |
| 24 | 3802 | 3820 | 3838 | 3856 | 3874 | 3892 | 3909 | 3927 | 3945 | 3962 | 2 4 | 5 7 | 9 | 11 12 | 14 16 |
| 25 | 3979 | 3997 | 4014 | 4031 | 4048 | 4065 | 4082 | 4099 | 4116 | 4133 | 2 3 | 5 7 | 9 | 10 12 | 14 15 |
| 26 | 4150 | 4166 | 4183 | 4200 | 4216 | 4232 | 4249 | 4265 | 4281 | 4298 | 2 3 | 5 7 | 8 | 10 11 | 13 15 |
| 27 | 4314 | 4330 | 4346 | 4362 | 4378 | 4393 | 4409 | 4425 | 4440 | 4456 | 2 3 | 5 6 | 8 | 9 11 | 13 14 |
| 28 | 4472 | 4487 | 4502 | 4518 | 4533 | 4548 | 4564 | 4579 | 4594 | 4609 | 2 3 | 5 6 | 8 | 9 11 | 12 14 |
| 29 | 4624 | 4639 | 4654 | 4669 | 4683 | 4698 | 4713 | 4728 | 4742 | 4757 | 1 3 | 4 6 | 7 | 9 10 | 12 13 |
| 30 | 4771 | 4786 | 4800 | 4814 | 4829 | 4843 | 4857 | 4871 | 4886 | 4900 | 1 3 | 4 6 | 7 | 9 10 | 11 13 |
| 31 | 4914 | 4928 | 4942 | 4955 | 4969 | 4983 | 4997 | 5011 | 5024 | 5038 | 1 3 | 4 6 | 7 | 8 10 | 11 12 |
| 32 | 5051 | 5065 | 5079 | 5092 | 5105 | 5119 | 5132 | 5145 | 5159 | 5172 | 1 3 | 4 5 | 7 | 8 9 | 11 12 |
| 33 | 5185 | 5198 | 5211 | 5224 | 5237 | 5250 | 5263 | 5276 | 5289 | 5302 | 1 3 | 4 5 | 6 | 8 9 | 10 12 |
| 34 | 5315 | 5328 | 5340 | 5353 | 5366 | 5378 | 5391 | 5403 | 5416 | 5428 | 1 3 | 4 5 | 6 | 8 9 | 10 11 |
| 35 | 5441 | 5453 | 5465 | 5478 | 5490 | 5502 | 5514 | 5527 | 5539 | 5551 | 1 2 | 4 5 | 6 | 7 9 | 10 11 |
| 36 | 5563 | 5575 | 5587 | 5599 | 5611 | 5623 | 5635 | 5647 | 5658 | 5670 | 1 2 | 4 5 | 6 | 7 8 | 10 11 |
| 37 | 5682 | 5694 | 5705 | 5717 | 5729 | 5740 | 5752 | 5763 | 5775 | 5786 | 1 2 | 3 5 | 6 | 7 8 | 9 10 |
| 38 | 5798 | 5809 | 5821 | 5832 | 5843 | 5855 | 5866 | 5877 | 5888 | 5899 | 1 2 | 3 5 | 6 | 7 8 | 9 10 |
| 39 | 5911 | 5922 | 5933 | 5944 | 5955 | 5966 | 5977 | 5988 | 5999 | 6010 | 1 2 | 3 4 | 5 | 7 8 | 9 10 |
| 40 | 6021 | 6031 | 6042 | 6053 | 6064 | 6075 | 6085 | 6096 | 6107 | 6117 | 1 2 | 3 4 | 5 | 6 8 | 9 10 |
| 41 | 6128 | 6138 | 6149 | 6160 | 6170 | 6180 | 6191 | 6201 | 6212 | 6222 | 1 2 | 3 4 | 5 | 6 7 | 8 9 |
| 42 | 6232 | 6243 | 6253 | 6263 | 6274 | 6284 | 6294 | 6304 | 6314 | 6325 | 1 2 | 3 4 | 5 | 6 7 | 8 9 |
| 43 | 6335 | 6345 | 6355 | 6365 | 6375 | 6385 | 6395 | 6405 | 6415 | 6425 | 1 2 | 3 4 | 5 | 6 7 | 8 9 |
| 44 | 6435 | 6444 | 6454 | 6464 | 6474 | 6484 | 6493 | 6503 | 6513 | 6522 | 1 2 | 3 4 | 5 | 6 7 | 8 9 |
| 45 | 6532 | 6542 | 6551 | 6561 | 6571 | 6580 | 6590 | 6599 | 6609 | 6618 | 1 2 | 3 4 | 5 | 6 7 | 8 9 |
| 46 | 6628 | 6637 | 6646 | 6656 | 6665 | 6675 | 6684 | 6693 | 6702 | 6712 | 1 2 | 3 4 | 5 | 6 7 | 7 8 |
| 47 | 6721 | 6730 | 6739 | 6749 | 6758 | 6767 | 6776 | 6785 | 6794 | 6803 | 1 2 | 3 4 | 5 | 5 6 | 7 8 |
| 48 | 6812 | 6821 | 6830 | 6839 | 6848 | 6857 | 6866 | 6875 | 6884 | 6893 | 1 2 | 3 4 | 4 | 5 6 | 7 8 |
| 49 | 6902 | 6911 | 6920 | 6928 | 6937 | 6946 | 6955 | 6964 | 6972 | 6981 | 1 2 | 3 4 | 4 | 5 6 | 7 8 |
| 50 | 6990 | 6998 | 7007 | 7016 | 7024 | 7033 | 7042 | 7050 | 7059 | 7067 | 1 2 | 3 3 | 4 | 5 6 | 7 8 |
| 51 | 7076 | 7084 | 7093 | 7101 | 7110 | 7118 | 7126 | 7135 | 7143 | 7152 | 1 2 | 3 3 | 4 | 5 6 | 7 8 |
| 52 | 7160 | 7168 | 7177 | 7185 | 7193 | 7202 | 7210 | 7218 | 7226 | 7235 | 1 2 | 2 3 | 4 | 5 6 | 7 7 |
| 53 | 7243 | 7251 | 7259 | 7267 | 7275 | 7284 | 7292 | 7300 | 7308 | 7316 | 1 2 | 2 3 | 4 | 5 6 | 6 7 |
| 54 | 7324 | 7332 | 7340 | 7348 | 7356 | 7364 | 7372 | 7380 | 7388 | 7396 | 1 2 | 2 3 | 4 | 5 6 | 6 7 |

LOGARITHMS.

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 12 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----|------|------|------|------|------|------|------|------|------|------|-----|---|---|---|---|---|---|---|
| 55 | 7404 | 7412 | 7419 | 7427 | 7435 | 7443 | 7451 | 7459 | 7466 | 7474 | 1 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |
| 56 | 7482 | 7490 | 7497 | 7505 | 7513 | 7520 | 7528 | 7536 | 7543 | 7551 | 1 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |
| 57 | 7559 | 7566 | 7574 | 7582 | 7589 | 7597 | 7604 | 7612 | 7619 | 7627 | 1 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |
| 58 | 7634 | 7642 | 7649 | 7657 | 7664 | 7672 | 7679 | 7686 | 7694 | 7701 | 1 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |
| 59 | 7709 | 7716 | 7723 | 7731 | 7738 | 7745 | 7752 | 7760 | 7767 | 7774 | 1 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |
| 60 | 7782 | 7789 | 7796 | 7803 | 7810 | 7818 | 7825 | 7832 | 7839 | 7846 | 1 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |
| 61 | 7853 | 7860 | 7868 | 7875 | 7882 | 7889 | 7896 | 7903 | 7910 | 7917 | 1 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |
| 62 | 7924 | 7931 | 7938 | 7945 | 7952 | 7959 | 7966 | 7973 | 7980 | 7987 | 1 1 | 2 | 3 | 3 | 4 | 5 | 6 | 6 |
| 63 | 7993 | 8000 | 8007 | 8014 | 8021 | 8028 | 8035 | 8041 | 8048 | 8055 | 1 1 | 2 | 3 | 3 | 4 | 5 | 6 | 6 |
| 64 | 8062 | 8069 | 8075 | 8082 | 8089 | 8096 | 8102 | 8109 | 8116 | 8122 | 1 1 | 2 | 3 | 3 | 4 | 5 | 6 | 6 |
| 65 | 8129 | 8136 | 8142 | 8149 | 8156 | 8162 | 8169 | 8176 | 8182 | 8189 | 1 1 | 2 | 3 | 3 | 4 | 5 | 6 | 6 |
| 66 | 8195 | 8202 | 8209 | 8215 | 8222 | 8228 | 8235 | 8241 | 8248 | 8254 | 1 1 | 2 | 3 | 3 | 4 | 5 | 6 | 6 |
| 67 | 8261 | 8267 | 8274 | 8280 | 8287 | 8293 | 8299 | 8306 | 8312 | 8319 | 1 1 | 2 | 3 | 3 | 4 | 5 | 6 | 6 |
| 68 | 8325 | 8331 | 8338 | 8344 | 8351 | 8357 | 8363 | 8370 | 8376 | 8382 | 1 1 | 2 | 3 | 3 | 4 | 4 | 5 | 6 |
| 69 | 8388 | 8395 | 8401 | 8407 | 8414 | 8420 | 8426 | 8432 | 8439 | 8445 | 1 1 | 2 | 2 | 3 | 4 | 4 | 5 | 6 |
| 70 | 8451 | 8457 | 8463 | 8470 | 8476 | 8482 | 8488 | 8494 | 8500 | 8506 | 1 1 | 2 | 2 | 3 | 4 | 4 | 5 | 6 |
| 71 | 8513 | 8519 | 8525 | 8531 | 8537 | 8543 | 8549 | 8555 | 8561 | 8567 | 1 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 72 | 8573 | 8579 | 8585 | 8591 | 8597 | 8603 | 8609 | 8615 | 8621 | 8627 | 1 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 73 | 8633 | 8639 | 8645 | 8651 | 8657 | 8663 | 8669 | 8675 | 8681 | 8686 | 1 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 74 | 8692 | 8698 | 8704 | 8710 | 8716 | 8722 | 8727 | 8733 | 8739 | 8745 | 1 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 75 | 8751 | 8756 | 8762 | 8768 | 8774 | 8779 | 8785 | 8791 | 8797 | 8802 | 1 1 | 2 | 2 | 3 | 3 | 4 | 5 | 5 |
| 76 | 8808 | 8814 | 8820 | 8825 | 8831 | 8837 | 8842 | 8848 | 8854 | 8859 | 1 1 | 2 | 2 | 3 | 3 | 4 | 5 | 5 |
| 77 | 8865 | 8871 | 8876 | 8882 | 8887 | 8893 | 8899 | 8904 | 8910 | 8915 | 1 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 78 | 8921 | 8927 | 8932 | 8938 | 8943 | 8949 | 8954 | 8960 | 8965 | 8971 | 1 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 79 | 8976 | 8982 | 8987 | 8993 | 8998 | 9004 | 9009 | 9015 | 9020 | 9025 | 1 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 80 | 9031 | 9036 | 9042 | 9047 | 9053 | 9058 | 9063 | 9069 | 9074 | 9079 | 1 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 81 | 9085 | 9090 | 9096 | 9101 | 9106 | 9112 | 9117 | 9122 | 9128 | 9133 | 1 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 82 | 9138 | 9143 | 9149 | 9154 | 9159 | 9165 | 9170 | 9175 | 9180 | 9186 | 1 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 83 | 9191 | 9196 | 9201 | 9206 | 9212 | 9217 | 9222 | 9227 | 9232 | 9238 | 1 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 84 | 9243 | 9248 | 9253 | 9258 | 9263 | 9269 | 9274 | 9279 | 9284 | 9289 | 1 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 85 | 9294 | 9299 | 9304 | 9309 | 9315 | 9320 | 9325 | 9330 | 9335 | 9340 | 1 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 86 | 9345 | 9350 | 9355 | 9360 | 9365 | 9370 | 9375 | 9380 | 9385 | 9390 | 1 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 87 | 9395 | 9400 | 9405 | 9410 | 9415 | 9420 | 9425 | 9430 | 9435 | 9440 | 0 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 88 | 9445 | 9450 | 9455 | 9460 | 9465 | 9469 | 9474 | 9479 | 9484 | 9489 | 0 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 89 | 9494 | 9499 | 9504 | 9509 | 9513 | 9518 | 9523 | 9528 | 9533 | 9538 | 0 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 90 | 9542 | 9547 | 9552 | 9557 | 9562 | 9566 | 9571 | 9576 | 9581 | 9586 | 0 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 91 | 9590 | 9595 | 9600 | 9605 | 9609 | 9614 | 9619 | 9624 | 9628 | 9633 | 0 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 92 | 9638 | 9643 | 9647 | 9652 | 9657 | 9661 | 9666 | 9671 | 9675 | 9680 | 0 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 93 | 9685 | 9689 | 9694 | 9699 | 9703 | 9708 | 9713 | 9717 | 9722 | 9727 | 0 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 94 | 9731 | 9736 | 9741 | 9745 | 9750 | 9754 | 9759 | 9763 | 9768 | 9773 | 0 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 95 | 9777 | 9782 | 9786 | 9791 | 9795 | 9800 | 9805 | 9809 | 9814 | 9818 | 0 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 96 | 9823 | 9827 | 9832 | 9836 | 9841 | 9845 | 9850 | 9854 | 9859 | 9863 | 0 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 97 | 9868 | 9872 | 9877 | 9881 | 9886 | 9890 | 9894 | 9899 | 9903 | 9908 | 0 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 98 | 9912 | 9917 | 9921 | 9926 | 9930 | 9934 | 9939 | 9943 | 9948 | 9952 | 0 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 99 | 9956 | 9961 | 9965 | 9969 | 9974 | 9978 | 9983 | 9987 | 9991 | 9996 | 0 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |

ANTILOGARITHMS.

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 12 | 34 | 5 | 6 | 7 | 8 | 9 |
|-----|------|------|------|------|------|------|------|------|------|------|-----|-----|---|---|---|---|---|
| '00 | 1000 | 1002 | 1005 | 1007 | 1009 | 1012 | 1014 | 1016 | 1019 | 1021 | 0 0 | 1 1 | 1 | 1 | 2 | 2 | 2 |
| '01 | 1023 | 1026 | 1028 | 1030 | 1033 | 1035 | 1038 | 1040 | 1042 | 1045 | 0 0 | 1 1 | 1 | 1 | 2 | 2 | 2 |
| '02 | 1047 | 1050 | 1052 | 1054 | 1057 | 1059 | 1062 | 1064 | 1067 | 1069 | 0 0 | 1 1 | 1 | 1 | 2 | 2 | 2 |
| '03 | 1072 | 1074 | 1076 | 1079 | 1081 | 1084 | 1086 | 1089 | 1091 | 1094 | 0 0 | 1 1 | 1 | 1 | 2 | 2 | 2 |
| '04 | 1096 | 1099 | 1102 | 1104 | 1107 | 1109 | 1112 | 1114 | 1117 | 1119 | 0 1 | 1 1 | 1 | 2 | 2 | 2 | 2 |
| '05 | 1122 | 1125 | 1127 | 1130 | 1132 | 1135 | 1138 | 1140 | 1143 | 1146 | 0 1 | 1 1 | 1 | 2 | 2 | 2 | 2 |
| '06 | 1148 | 1151 | 1153 | 1156 | 1159 | 1161 | 1164 | 1167 | 1169 | 1172 | 0 1 | 1 1 | 1 | 2 | 2 | 2 | 2 |
| '07 | 1175 | 1178 | 1180 | 1183 | 1186 | 1189 | 1191 | 1194 | 1197 | 1199 | 0 1 | 1 1 | 1 | 2 | 2 | 2 | 2 |
| '08 | 1202 | 1205 | 1208 | 1211 | 1213 | 1216 | 1219 | 1222 | 1225 | 1227 | 0 1 | 1 1 | 1 | 2 | 2 | 2 | 3 |
| '09 | 1230 | 1233 | 1236 | 1239 | 1242 | 1245 | 1247 | 1250 | 1253 | 1256 | 0 1 | 1 1 | 1 | 2 | 2 | 2 | 3 |
| '10 | 1259 | 1262 | 1265 | 1268 | 1271 | 1274 | 1276 | 1279 | 1282 | 1285 | 0 1 | 1 1 | 1 | 2 | 2 | 2 | 3 |
| '11 | 1288 | 1291 | 1294 | 1297 | 1300 | 1303 | 1306 | 1309 | 1312 | 1315 | 0 1 | 1 1 | 2 | 2 | 2 | 2 | 3 |
| '12 | 1318 | 1321 | 1324 | 1327 | 1330 | 1334 | 1337 | 1340 | 1343 | 1346 | 0 1 | 1 1 | 2 | 2 | 2 | 2 | 3 |
| '13 | 1349 | 1352 | 1355 | 1358 | 1361 | 1365 | 1368 | 1371 | 1374 | 1377 | 0 1 | 1 1 | 2 | 2 | 2 | 3 | 3 |
| '14 | 1380 | 1384 | 1387 | 1390 | 1393 | 1396 | 1400 | 1403 | 1406 | 1409 | 0 1 | 1 1 | 2 | 2 | 2 | 3 | 3 |
| '15 | 1413 | 1416 | 1419 | 1422 | 1426 | 1429 | 1432 | 1435 | 1439 | 1442 | 0 1 | 1 1 | 2 | 2 | 2 | 3 | 3 |
| '16 | 1445 | 1449 | 1452 | 1455 | 1459 | 1462 | 1466 | 1469 | 1472 | 1476 | 0 1 | 1 1 | 2 | 2 | 2 | 3 | 3 |
| '17 | 1479 | 1483 | 1486 | 1489 | 1493 | 1496 | 1500 | 1503 | 1507 | 1510 | 0 1 | 1 1 | 2 | 2 | 2 | 3 | 3 |
| '18 | 1514 | 1517 | 1521 | 1524 | 1528 | 1531 | 1535 | 1538 | 1542 | 1545 | 0 1 | 1 1 | 2 | 2 | 2 | 3 | 3 |
| '19 | 1549 | 1552 | 1556 | 1560 | 1563 | 1567 | 1570 | 1574 | 1578 | 1581 | 0 1 | 1 1 | 2 | 2 | 3 | 3 | 3 |
| '20 | 1585 | 1589 | 1592 | 1596 | 1600 | 1603 | 1607 | 1611 | 1614 | 1618 | 0 1 | 1 1 | 2 | 2 | 3 | 3 | 3 |
| '21 | 1622 | 1626 | 1629 | 1633 | 1637 | 1641 | 1644 | 1648 | 1652 | 1656 | 0 1 | 1 2 | 2 | 2 | 3 | 3 | 3 |
| '22 | 1660 | 1663 | 1667 | 1671 | 1675 | 1679 | 1683 | 1687 | 1690 | 1694 | 0 1 | 1 2 | 2 | 2 | 3 | 3 | 3 |
| '23 | 1698 | 1702 | 1706 | 1710 | 1714 | 1718 | 1722 | 1726 | 1730 | 1734 | 0 1 | 1 2 | 2 | 2 | 3 | 3 | 4 |
| '24 | 1738 | 1742 | 1746 | 1750 | 1754 | 1758 | 1762 | 1766 | 1770 | 1774 | 0 1 | 1 2 | 2 | 2 | 3 | 3 | 4 |
| '25 | 1778 | 1782 | 1786 | 1791 | 1795 | 1799 | 1803 | 1807 | 1811 | 1816 | 0 1 | 1 2 | 2 | 2 | 3 | 3 | 4 |
| '26 | 1820 | 1824 | 1828 | 1832 | 1837 | 1841 | 1845 | 1849 | 1854 | 1858 | 0 1 | 1 2 | 2 | 3 | 3 | 3 | 4 |
| '27 | 1862 | 1866 | 1871 | 1875 | 1879 | 1884 | 1888 | 1892 | 1897 | 1901 | 0 1 | 1 2 | 2 | 3 | 3 | 3 | 4 |
| '28 | 1905 | 1910 | 1914 | 1919 | 1923 | 1928 | 1932 | 1936 | 1941 | 1945 | 0 1 | 1 2 | 2 | 3 | 3 | 4 | 4 |
| '29 | 1950 | 1954 | 1959 | 1963 | 1968 | 1972 | 1977 | 1982 | 1986 | 1991 | 0 1 | 1 2 | 2 | 3 | 3 | 4 | 4 |
| '30 | 1995 | 2000 | 2004 | 2009 | 2014 | 2018 | 2023 | 2028 | 2032 | 2037 | 0 1 | 1 2 | 2 | 3 | 3 | 4 | 4 |
| '31 | 2042 | 2046 | 2051 | 2056 | 2061 | 2065 | 2070 | 2075 | 2080 | 2084 | 0 1 | 1 2 | 2 | 3 | 3 | 4 | 4 |
| '32 | 2089 | 2094 | 2099 | 2104 | 2109 | 2113 | 2118 | 2123 | 2128 | 2133 | 0 1 | 1 2 | 2 | 3 | 3 | 4 | 4 |
| '33 | 2138 | 2143 | 2148 | 2153 | 2158 | 2163 | 2168 | 2173 | 2178 | 2183 | 0 1 | 1 2 | 2 | 3 | 3 | 4 | 4 |
| '34 | 2188 | 2193 | 2198 | 2203 | 2208 | 2213 | 2218 | 2223 | 2228 | 2234 | 1 1 | 2 2 | 3 | 3 | 4 | 4 | 5 |
| '35 | 2239 | 2244 | 2249 | 2254 | 2259 | 2265 | 2270 | 2275 | 2280 | 2286 | 1 1 | 2 2 | 3 | 3 | 4 | 4 | 5 |
| '36 | 2291 | 2296 | 2301 | 2307 | 2312 | 2317 | 2323 | 2328 | 2333 | 2339 | 1 1 | 2 2 | 3 | 3 | 4 | 4 | 5 |
| '37 | 2344 | 2350 | 2355 | 2360 | 2366 | 2371 | 2377 | 2382 | 2388 | 2393 | 1 1 | 2 2 | 3 | 3 | 4 | 4 | 5 |
| '38 | 2399 | 2404 | 2410 | 2415 | 2421 | 2427 | 2432 | 2438 | 2443 | 2449 | 1 1 | 2 2 | 3 | 3 | 4 | 4 | 5 |
| '39 | 2455 | 2460 | 2466 | 2472 | 2477 | 2483 | 2489 | 2495 | 2500 | 2506 | 1 1 | 2 2 | 3 | 3 | 4 | 4 | 5 |
| '40 | 2512 | 2518 | 2523 | 2529 | 2535 | 2541 | 2547 | 2553 | 2559 | 2564 | 1 1 | 2 2 | 3 | 4 | 4 | 5 | 5 |
| '41 | 2570 | 2576 | 2582 | 2588 | 2594 | 2600 | 2606 | 2612 | 2618 | 2624 | 1 1 | 2 2 | 3 | 4 | 4 | 5 | 5 |
| '42 | 2630 | 2636 | 2642 | 2649 | 2655 | 2661 | 2667 | 2673 | 2679 | 2685 | 1 1 | 2 2 | 3 | 4 | 4 | 5 | 6 |
| '43 | 2692 | 2698 | 2704 | 2710 | 2716 | 2723 | 2729 | 2735 | 2742 | 2748 | 1 1 | 2 3 | 3 | 4 | 4 | 5 | 6 |
| '44 | 2754 | 2761 | 2767 | 2773 | 2780 | 2786 | 2793 | 2799 | 2805 | 2812 | 1 1 | 2 3 | 3 | 4 | 4 | 5 | 6 |
| '45 | 2818 | 2825 | 2831 | 2838 | 2844 | 2851 | 2858 | 2864 | 2871 | 2877 | 1 1 | 2 3 | 3 | 4 | 5 | 5 | 6 |
| '46 | 2884 | 2891 | 2897 | 2904 | 2911 | 2917 | 2924 | 2931 | 2938 | 2944 | 1 1 | 2 3 | 3 | 4 | 5 | 5 | 6 |
| '47 | 2951 | 2958 | 2965 | 2972 | 2979 | 2985 | 2992 | 2999 | 3006 | 3013 | 1 1 | 2 3 | 3 | 4 | 5 | 5 | 6 |
| '48 | 3020 | 3027 | 3034 | 3041 | 3048 | 3055 | 3062 | 3069 | 3076 | 3083 | 1 1 | 2 3 | 4 | 4 | 5 | 6 | 6 |
| '49 | 3090 | 3097 | 3105 | 3112 | 3119 | 3126 | 3133 | 3141 | 3148 | 3155 | 1 1 | 2 3 | 4 | 4 | 5 | 6 | 6 |

ANTILOGARITHMS.

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 12 | 34 | 5 | 6 | 7 | 8 | 9 |
|-----|------|------|------|------|------|------|------|------|------|------|-----|-----|----|----|----|----|----|
| *50 | 3162 | 3170 | 3177 | 3184 | 3192 | 3199 | 3206 | 3214 | 3221 | 3228 | 1 1 | 2 3 | 4 | 4 | 5 | 6 | 7 |
| *51 | 3236 | 3243 | 3251 | 3258 | 3266 | 3273 | 3281 | 3289 | 3296 | 3304 | 1 2 | 2 3 | 4 | 5 | 5 | 6 | 7 |
| *52 | 3311 | 3319 | 3327 | 3334 | 3342 | 3350 | 3357 | 3365 | 3373 | 3381 | 1 2 | 2 3 | 4 | 5 | 5 | 6 | 7 |
| *53 | 3388 | 3396 | 3404 | 3412 | 3420 | 3428 | 3436 | 3443 | 3451 | 3459 | 1 2 | 2 3 | 4 | 5 | 6 | 6 | 7 |
| *54 | 3467 | 3475 | 3483 | 3491 | 3499 | 3508 | 3516 | 3524 | 3532 | 3540 | 1 2 | 2 3 | 4 | 5 | 6 | 6 | 7 |
| *55 | 3548 | 3556 | 3565 | 3573 | 3581 | 3589 | 3597 | 3606 | 3614 | 3622 | 1 2 | 2 3 | 4 | 5 | 6 | 7 | 7 |
| *56 | 3631 | 3639 | 3648 | 3656 | 3664 | 3673 | 3681 | 3690 | 3698 | 3707 | 1 2 | 3 3 | 4 | 5 | 6 | 7 | 8 |
| *57 | 3715 | 3724 | 3733 | 3741 | 3750 | 3758 | 3767 | 3776 | 3784 | 3793 | 1 2 | 3 3 | 4 | 5 | 6 | 7 | 8 |
| *58 | 3802 | 3811 | 3819 | 3828 | 3837 | 3846 | 3855 | 3864 | 3873 | 3882 | 1 2 | 3 4 | 4 | 5 | 6 | 7 | 8 |
| *59 | 3890 | 3899 | 3908 | 3917 | 3926 | 3936 | 3945 | 3954 | 3963 | 3972 | 1 2 | 3 4 | 5 | 5 | 6 | 7 | 8 |
| *60 | 3981 | 3990 | 3999 | 4009 | 4018 | 4027 | 4036 | 4046 | 4055 | 4064 | 1 2 | 3 4 | 5 | 6 | 6 | 7 | 8 |
| *61 | 4074 | 4083 | 4093 | 4102 | 4111 | 4121 | 4130 | 4140 | 4150 | 4159 | 1 2 | 3 4 | 5 | 6 | 7 | 8 | 9 |
| *62 | 4169 | 4178 | 4188 | 4198 | 4207 | 4217 | 4227 | 4236 | 4246 | 4256 | 1 2 | 3 4 | 5 | 6 | 7 | 8 | 9 |
| *63 | 4266 | 4276 | 4285 | 4295 | 4305 | 4315 | 4325 | 4335 | 4345 | 4355 | 1 2 | 3 4 | 5 | 6 | 7 | 8 | 9 |
| *64 | 4365 | 4375 | 4385 | 4395 | 4406 | 4416 | 4426 | 4436 | 4446 | 4457 | 1 2 | 3 4 | 5 | 6 | 7 | 8 | 9 |
| *65 | 4467 | 4477 | 4487 | 4498 | 4508 | 4519 | 4529 | 4539 | 4550 | | 1 2 | 3 4 | 5 | 6 | 7 | 8 | 9 |
| *66 | 4571 | 4581 | 4592 | 4603 | 4613 | 4624 | 4634 | 4645 | 4656 | 4667 | 1 2 | 3 4 | 5 | 6 | 7 | 9 | 10 |
| *67 | 4677 | 4688 | 4699 | 4710 | 4721 | 4732 | 4742 | 4753 | 4764 | 4775 | 1 2 | 3 4 | 5 | 7 | 8 | 9 | 10 |
| *68 | 4786 | 4797 | 4808 | 4819 | 4831 | 4842 | 4853 | 4864 | 4875 | 4887 | 1 2 | 3 4 | 6 | 7 | 8 | 9 | 10 |
| *69 | 4898 | 4909 | 4920 | 4932 | 4943 | 4955 | 4966 | 4977 | 4989 | 5000 | 1 2 | 3 5 | 6 | 7 | 8 | 9 | 10 |
| *70 | 5012 | 5023 | 5035 | 5047 | 5058 | 5070 | 5082 | 5093 | 5105 | 5117 | 1 2 | 4 5 | 6 | 7 | 8 | 9 | 11 |
| *71 | 5129 | 5140 | 5152 | 5164 | 5176 | 5188 | 5200 | 5212 | 5224 | 5236 | 1 2 | 4 5 | 6 | 7 | 8 | 10 | 11 |
| *72 | 5248 | 5260 | 5272 | 5284 | 5297 | 5309 | 5321 | 5333 | 5346 | 5358 | 1 2 | 4 5 | 6 | 7 | 9 | 10 | 11 |
| *73 | 5370 | 5383 | 5395 | 5408 | 5420 | 5433 | 5445 | 5458 | 5470 | 5483 | 1 3 | 4 5 | 6 | 8 | 9 | 10 | 11 |
| *74 | 5495 | 5508 | 5521 | 5534 | 5546 | 5559 | 5572 | 5585 | 5598 | 5610 | 1 3 | 4 5 | 6 | 8 | 9 | 10 | 12 |
| *75 | 5623 | 5636 | 5649 | 5662 | 5675 | 5689 | 5702 | 5715 | 5728 | 5741 | 1 3 | 4 5 | 7 | 8 | 9 | 10 | 12 |
| *76 | 5754 | 5768 | 5781 | 5794 | 5808 | 821 | 5834 | 5848 | 5861 | 5875 | 1 3 | 4 5 | 7 | 8 | 9 | 11 | 12 |
| *77 | 5888 | 5902 | 5916 | 5929 | 5943 | 5957 | 5970 | 5984 | 5998 | 6012 | 1 3 | 4 5 | 7 | 8 | 10 | 11 | 12 |
| *78 | 6026 | 6039 | 6053 | 6067 | 6081 | 6095 | 6109 | 6124 | 6138 | 6152 | 1 3 | 4 6 | 7 | 8 | 10 | 11 | 13 |
| *79 | 6166 | 6180 | 6194 | 6209 | 6223 | 6237 | 6252 | 6266 | 6281 | 6295 | 1 3 | 4 6 | 7 | 9 | 10 | 11 | 13 |
| *80 | 6310 | 6324 | 6339 | 6353 | 6368 | 6383 | 6397 | 6412 | 6427 | 6442 | 1 3 | 4 6 | 7 | 9 | 10 | 12 | 13 |
| *81 | 6457 | 6471 | 6486 | 6501 | 6516 | 6531 | 6546 | 6561 | 6577 | 6592 | 2 3 | 5 6 | 8 | 9 | 11 | 12 | 14 |
| *82 | 6607 | 6622 | 6637 | 6653 | 6668 | 6683 | 6699 | 6714 | 6730 | 6745 | 2 3 | 5 6 | 8 | 9 | 11 | 12 | 14 |
| *83 | 6761 | 6776 | 6792 | 6808 | 6823 | 6839 | 6855 | 6871 | 6887 | 6902 | 2 3 | 5 6 | 8 | 9 | 11 | 13 | 14 |
| *84 | 6918 | 6934 | 6950 | 6966 | 6982 | 6998 | 7015 | 7031 | 7047 | 7063 | 2 3 | 5 6 | 8 | 10 | 11 | 13 | 15 |
| *85 | 7079 | 7096 | 7112 | 7129 | 7145 | 7161 | 7178 | 7194 | 7211 | 7228 | 2 3 | 5 7 | 8 | 10 | 12 | 13 | 15 |
| *86 | 7244 | 7261 | 7278 | 7295 | 7311 | 7328 | 7345 | 7362 | 7379 | 7396 | 2 3 | 5 7 | 8 | 10 | 12 | 13 | 15 |
| *87 | 7413 | 7430 | 7447 | 7464 | 7482 | 7499 | 7516 | 7534 | 7551 | 7568 | 2 3 | 5 7 | 9 | 10 | 12 | 14 | 16 |
| *88 | 7586 | 7603 | 7621 | 7638 | 7656 | 7674 | 7691 | 7709 | 7727 | 7745 | 2 4 | 5 7 | 9 | 11 | 12 | 14 | 16 |
| *89 | 7762 | 7780 | 7798 | 7816 | 7834 | 7852 | 7870 | 7889 | 7907 | 7925 | 2 4 | 5 7 | 9 | 11 | 12 | 14 | 16 |
| *90 | 7943 | 7962 | 7980 | 7998 | 8017 | 8035 | 8054 | 8072 | 8091 | 8110 | 2 4 | 6 7 | 9 | 11 | 13 | 15 | 17 |
| *91 | 8128 | 8147 | 8166 | 8185 | 8204 | 8222 | 8241 | 8260 | 8279 | 8299 | 2 4 | 6 8 | 9 | 11 | 13 | 15 | 17 |
| *92 | 8318 | 8337 | 8356 | 8375 | 8395 | 8414 | 8433 | 8453 | 8472 | 8492 | 2 4 | 6 8 | 10 | 12 | 14 | 15 | 17 |
| *93 | 8511 | 8531 | 8551 | 8570 | 8590 | 8610 | 8630 | 8650 | 8670 | 8690 | 2 4 | 6 8 | 10 | 12 | 14 | 16 | 18 |
| *94 | 8710 | 8730 | 8750 | 8770 | 8790 | 8810 | 8831 | 8851 | 8872 | 8892 | 2 4 | 6 8 | 10 | 12 | 14 | 16 | 18 |
| *95 | 8913 | 8933 | 8954 | 8974 | 8995 | 9016 | 9036 | 9057 | 9078 | 9099 | 2 4 | 6 8 | 10 | 12 | 15 | 17 | 19 |
| *96 | 9120 | 9141 | 9162 | 9183 | 9204 | 9226 | 9247 | 9268 | 9290 | 9311 | 2 4 | 6 8 | 11 | 13 | 15 | 17 | 19 |
| *97 | 9333 | 9354 | 9376 | 9397 | 9419 | 9441 | 9463 | 9484 | 9506 | 9528 | 2 4 | 7 9 | 11 | 13 | 15 | 17 | 20 |
| *98 | 9550 | 9572 | 9594 | 9616 | 9638 | 9661 | 9683 | 9705 | 9727 | 9750 | 2 4 | 7 9 | 11 | 13 | 16 | 18 | 20 |
| *99 | 9772 | 9795 | 9817 | 9840 | 9863 | 9886 | 9908 | 9931 | 9954 | 9977 | 2 5 | 7 9 | 11 | 14 | 16 | 18 | 20 |

INDEX

- Activity, or rate of doing work (P), 1
——— measurement of, by ammeter and voltmeter, 12
- Alternating-current circuit, 113
——— shape of wave, 114
——— value of B.E.M.F. in finite solenoid, 131
——— chemical effect, 115
——— thermal effect, 116
——— virtual value of, 117
——— magnetic effect of, 118-144
——— B.E.M.F. of self-induction, 120, 127
——— power in circuit, 125
——— lag of, 123
——— lag of, with small B.E.M.F., 125
——— ——— medium B.E.M.F., 126
——— ——— large B.E.M.F., 127
- Ammeter, winding of coil of, 85
- Ampère-turns magnetizing force, 70
——— required for armature, 77
——— ——— air gaps, 77
——— ——— magnet limbs, 77
——— ——— yoke, 77
——— ——— compounding, 90
——— by transformer iron, 155
——— ——— voltmeters and ammeters, 85
- Ampères required by tramcar, 109
- Arc lamps, 33
- Area of armature iron, 75
——— air gaps, 75
——— magnet limb, 75
- Armature, reactions in, 86
——— wire, pull on, 48
——— total pull on, 49
——— power to drive, 50
——— winding of, 55
——— losses in, 55
——— temperature rise of, 56
——— hysteresis loss, calculation of, 57
——— eddy-current loss, calculation of, 57
——— cross magnetizing wires on, 88
——— demagnetizing wires on, 88

- β , the magnetic density, 65, 68
- B.E.M.F. of secondary cell, 28
 - self-induction, 120
 - , magnitude of, 127
- Capacity in alternating-current circuit, 160
 - in parallel with circuit, 170
 - in series with circuit, 169
 - , current due to, 165
 - , E.M.F. due to, 167
 - of concentric cables, 171
 - size of 1 farad condenser, 169
 - , specific inductive, values of, 168
- Circuit, simple, 1
 - , series, 3
 - , parallel, 4
 - , multiple wire, 15
 - magnetic, compared with electric, 72
- Coil winding, series class, 82
 - , shunt class, 83
- Condenser, use of, in alternating-current circuit, 169
 - , size of 1 farad, 169
- Conductivity (m), 2
- Conductance, 2
- Cross magnetizing wires on armature, 88
- Demagnetizing wires on armature, 88
- Distribution, 15
- Distributor, definition of, 17
- Drop of volts in distributor, 16
 - in feeder, 16
 - in armature, series, 36
 - , shunt, 43
 - , compound, long shunt, 44
 - , short shunt, 45
 - in transformer primary, 149
 - secondary, 149
 - due to B.E.M.F. of secondary, 150
 - due to magnetic leakage, 150
- Dynamo, formula for E.M.F. of, 35
 - , P.D. at terminals, 36
 - , separately excited, 40
 - , series, 41
 - , shunt, 42
 - , compound, long shunt, 43
 - , short shunt, 44
 - , calculation of amount of series winding, 86-92
- Economical size of feeders, 19
- Eddy-current loss in armature, 57
 - in transformer, 150
 - , current to balance, 136
- Efficiency of distribution and transmission, 95
 - magneto machine, 97

- Efficiency of series dynamo, 97
 - shunt dynamo, 97
 - compound dynamo, short shunt, 98.
 - , long shunt, 99
 - series motor, 99
 - shunt motor, 100
 - compound motor, short shunt, 100
 - , long shunt, 101
 - secondary battery, 101
 - combined plant, 101
 - transformer, all-day, 156
- Electro-motive force, E.M.F. defined, 1
- E.M.F. active (ϵ), 27
 - back (ϵ), 27
 - total (E), 27
- Feeder, definition of, 17
 - , economical size of, 19
 - , working current in, 19
- Field-magnet winding, series, 39-82
 - , shunt, 83
 - , compound, 90
- Five-wire system, 23
- Gap, air or iron to iron space, area of, 75
- Gas-engine, cost of gas consumed by, 181
- H, strength of magnetic field, or magnetizing force, 67
- Heating of armature, 56
 - voltmeter, 102
- Hysteresis loss in transformer, 151
 - in armature, 57
 - , effect of on shape of current curve in alternating-current circuit, 137
 - in cyclic change of magnetization, 138
- Impedance coils, 145
 - , why open iron circuit type, 140
- Insulation, electric resistance of, 10
 - , magnetic, 73
- Impressed E.M.F. alternating, how found, 122
- Kelvin's law of economy, 19
- Kennelley's safety rule, 21
- Leakage of magnetic flux, 73
 - coefficient (ν), 74
 - drop in transformer, 150
- Length, effective, of solenoid, 132
 - of magnetic path through armature, 75
 - air gaps, 75
 - magnet limbs, 75
 - yoke, 75

- Magnet coil, series, size of wire for, 82
- , shunt, size of wire for, 83
- Magnetic properties of iron, 69
 - circuit compared with electric, 72
 - of dynamo, analysis of, 75
 - of finite solenoid, 131
- Magnetizing force (H) physical measure, 68
- (H) ampère-turns, 70
- Motors, 31
 - , armature reactions in, 92
 - , compound winding of, 93
 - , B.E.M.F. of, 51
- Ohm's law for continuous currents, 1
- Ohmmeter, connections of, 14
- Open iron circuit transformer, 143
- Periodicity or frequency, 114
- Permeability (μ), 66
- Potential difference (P.D.) definition of, 1
- Power to drive armature, 50
- Power or activity, (P), 2
- Power factor (ϕ) in alternating-current circuit, 126
- Powermeter, connections of, 14
- Powermeter for alternating current circuit, 170
- Prime mover costs, 181
- Pull on armature wire, 48
- Quantity-efficiency of secondary battery, 101
- Railway, electric, 106
 - tractive force required, 106
 - power required, 107
 - pull to get up speed, 109
 - E.M.F. required, 107
 - current, 109
 - shunt motor on, 110
 - compound motor on, 111
 - series motor on, 112
- Rate of doing work, or activity, 1
- Regulation of speed of motor, 174
- Reluctance magnetic (\mathfrak{K}), 72
 - of air space surrounding coil, 131
- Resistance to start a motor, 174
 - in series with motor, 176
 - in shunt coil of motor, 177
 - calculation of series circuit, 2
 - parallel circuit, 5
 - of insulation, 10
 - measurement of by ammeter and voltmeter, 12
- Resistivity (ρ) definition of, 2
- Safety rules, 21
- Secondary battery, E.M.F. curves, 28

Solenoid, infinite, magnetic properties of, 67

———, finite, effective length of, 131

Specific inductive capacity, values of, 168

Steam engine, cost of fuel in, 181

Temperature rise, Esson's rules, 55

Transformer, balancing of secondary output in closed iron circuit type, 141

———, in open iron type, 143

———, relationship between power factor and output, 143

———, design of, 148

———, eddy-current loss in, 150

———, hysteresis loss in, 151

———, all-day efficiency of, 156

Voltmeter, size of wire for coil of, 85

———, resistance of, 102

———, extra coil for, 104

———, effects of temperature on, 102

———, heating error in, 102

———, temperature error in, 102

———, constant of, at any temperature, 105

Work, rate of doing, 1

Watt, defined, 2

THE END.



UNIVERSITY OF CALIFORNIA LIBRARY
BERKELEY

Return to desk from which borrowed.

This book is DUE on the last date stamped below.

ENGINEERING LIBRARY

OCT .8 1948

JUL 5 1950 *KA*

YB 24105

45

6

77

T.T
168
J6

Engineering
Library

74023

